THE USE OF BOUNDING WELLS
TO COUNTERACT THE EFFECTS OF
GRAVITY IN DIPPING AQUIFERS

A Thesis

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TABLE OF CONTENTS

TABLE OF CONTENTS CONT'D

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES CONT'D

ABSTRACT

Previous studies concerning the cyclic storage of fresh water in saline aquifers have pointed out the deleterious effects of pre-existing groundwater movement and aquifer dip. The theoretical feasibility of using a system of peripheral bounding wells around a proposed storage area to negate the effects of gross fluid movement has been previously shown. Such a bounding well field would contain both injection and production wells. The present study experimentally validated the previous theoretical work and expanded it to include the use of bounding wells to partially negate the effects of gravity in dipping aquifers.

It was experimentally shown that bounding wells can be used in a dipping system to effectively prevent the gravitational migration of an injected fluid whose density differs from that of the original aquifer fluid. A computer program developed to predict the required bounding well flow rates was shown to accurately predict the behavior of the physical system. It was demonstrated that bounding well flow rates can be choosen such that a balance exists between injection and production volumes, so that none of the fluid produced by the bounding wells will reach the

biosphere. One of the parameters shown to affect bounding well flow rates is radius of the injected volume of fluid, hence the required bounding well rates change continuously during injection and production of the stored fluid.

CHAPTER I

INTRODUCTION

The process of storing fresh water in a saline aquifer is a cyclic process which involves injecting fresh water when there is a surplus, and producing it from the same wells when there is a drought or peak demand. The advantages of subsurface storage over surface storage, such as steel tanks or surface reservoirs are:

(1) The prohibitive cost of surface storage

(2) Reservoirs are open to pollution

(3) Loss of water through evaporation

(4) Damage to reservoirs by natural causes

The feasibility of fresh water storage in underground saline aquifers has been studied by several investigators: Esmail[1], Esmail and Kimbler[2], Kumar and Kimbler[3], Francis[4], Painter[5], Esmail[6], Kimbler, Kazmann and Whitehead[7], D'Amico[8], Langhetee[9], Tate[10], Agrawal[11], and Whitehead and Langhetee[12].

In the storage process there are several parameters that are of importance. These are: the dispersivity of the aquifer, the density difference between injected fluid and native fluid, the dip of the aquifer, pre-existing ground water movement (flux), the transmissivity of the aquifer, the homogeneity of the aquifer, rate of injection, and duration of storage.

The ratio of the volume of usable water recoverable to the volume of water injected has been defined by previous investigators as recovery efficiency. Esmail[6] found that pre-existing groundwater movement significantly decreases recovery efficiency. The pre-existing groundwater movement will cause the injected bubble of stored water to move with it. This causes the native fluid to "breakthrough" on the upstream edge of the injected bubble before it does on the downstream edge. Langehetee[9] undertook the theoretical investigation of the use of bounding wells to offset the effects of flux in horizontal aquifers. This method involves the placing of bounding wells around any size storage area, and operating these wells in a manner that will reduce all pre-existing potentiometric gradients within the proposed storage area to zero. Thus a bubble of fresh water injected into the storage area, will have no tendency to migrate.

Esmail[6], Painter[5], Francis[4], Tate[10], and Agrawal[11] all investigated the effect of dip on storage of fresh water in saline aquifers. Recovery efficiencies were reduced due to the gravitational effects associated with dip. If the injected fluid is less dense than the native fluid, it will migrate up dip. If it is greater, then the injected bubble will migrate down dip. In either instance, one edge of the bubble will breakthrough before the other, thus reducing recovery efficiency.

The purpose of this investigation was twofold: (1) to experimentally test the theoretical predictions of Langhetee[9], (2) to investigate the feasibility of using bounding wells to at least partially offset the deletereous effects of dip. This second objective requires that the mathematical model proposed by Whitehead and Langhetee[12] be extended and that the resulting theoretical treatment be experimentally tested in a physical system.

CHAPTER II

REVIEW OF PREVIOUS INVESTIGATIONS

The earliest attempt at the storage of fresh water in a saline aquifer was made by Cederstrom[13] in 1947. Esmail and Kimbler[2] (1967), Kumar and Kimbler[3] (1969) investigated the technical feasibility of storing fresh water in a horizontal saline aquifer in which there was no pre-existing ground water movement. They found that the mixing between miscible fluids retarded gravitational segregation. They also concluded that for the conditions studied the storage of fresh water in a horizontal saline aquifer was technically feasible. This work was extended by Whitehead to include the use of well fields under similar aquifer conditions.

As early as 1964, Hapaz and Bear[15] reported that the subsurface storage of fresh water could be unsuccessful if there was pre-existing movement of the native water. Esmail[6] undertook an experimental investigation of flux and its effect on recovery efficiency in a thin homogeneous mini-aquifer. He found that the recovery efficiency dedreased when flux was present. This decrease was a result of the migration of the injected fluid away from the well bore and was proportional to the magnitude of the flux.

It was apparent from this work that a pre-existing ground water movement of even a fraction of a foot per day would render subsurface storage uneconomical where long storage times were required.

In view of the harmful effect of flux as reported by Esmail[6], Langhetee[9] undertook a theoretical study aimed at counteracting this effect in horizontal aquifers. This study proposed the use of a system of bounding wells arbitrarily placed around a specified storage area and operated at predetermined rates. A number of the wells would produce water and the remaining wells would reinject this water into the aquifer, thereby reducing the pre-existing gradient in the storage area to zero. The computational procedure originally proposed by Langhetee[9] and subsequently modified by Whitehead and Langhetee[12] predicts the required bounding well flow rates. Two significant findings were (1) that these predicted flow rates produced a balanced system. Since all produced water was reinjected into the same aquifer no environmental problems were encountered; (2) the minimum distance from the center of the desired storage radius to the nearest bounding well should not be less than four thirds of the storage radius. No experimental verification was attempted by Langhetee since the proposed computational procedure was applicable only to aquifers of infinite areal extent.

Esmail[6], Painter[5], D'Amico[8] and Tate[10] investigated the effects of dip on the fresh water storage process. As with the horizontal studies concerning flux, the recovery efficiencies decreased due to injected fluid migration away from the injection-production well.

Painter[5] described the effect of dip on the configuration, migration and recovery of an injected fluid in a dipping aquifer penetrated by a single well. He proposed a mathematical solution and verified it by experiments in a large consolidated sandstone mini-aquifer. He concluded that the movement of the injected fluid front in a dipping system sufficiently thin to be considered two-dimensional may be approximated by combining the steady-state solution to the diffusivity equation with an empirically weighted term involving dip angle, injection rate, time, density and a system pore volume base upon the radius to a circular isopotential surrounding the storage area.

D'Amico, working on a three-dimensional mini-aquifer, verified Painter's solutions. Using an injection fluid more dense than the native fluid and dip angles ranging from five to twenty degrees he concluded that:

(1) as dip angle and size of the injected bubble increased,the downdip frontal velocity at the floor of the aquifer increased.

(2) the updip frontal velocity at the floor decreased with injected volume and eventually stabilized.

(3) the injected fluid configuration at the roof of the aquifer remained circular and concentric with the injection well.

Tate[10] extended the work of D'Amico and reached similar conclusions.

The above investigations show that although aquifer storage is feasible in the absence of appreciable dip or flux, either of these factors may tend to be highly detrimental. For successful field application of the process it may be necessary that these effects be negated since most aquifers otherwise suitable, will pocess dip, flux or both. Although the work of Langhetee[9] suggested a possible solution for the case of flux, his solution has not yet been experimentally demonstrated to be feasible and no remedy has been suggested to offset the effects of dip.

CHAPTER III

THEORY

3.1 Horizontal System with Flux

In a horizontal, homogeneous, isotropic aquifer of infinite areal extent which is saturated with a liquid at rest, the flow potential at every point is constant. If a well is located at point $(x_i y_i)$ with respect to an artibtary coordinate system, and operated at a flow rate q_i, then the steady-state potential at any point (x,y) is given by[16]:

$$\Phi(x,y) = -\frac{\mu}{4\pi\ kh} q_i \ln[(x-x_i)^2+(y-y_i)^2]+C_1 \quad . \ . \ . \ .3.1$$

$\Phi(x,y)$ = potential at any point, (atmospheres)

μ = viscosity, (centipoises)

k = permeability, (darcies)

h = thickness, (centimeters)

q_i = injection rate, (cubic centimeters per sec.) ($q_i > 0$ injector , $q_i < 0$ producer)

C_1 = an arbitrary level setting constant, (atmospheres)

If there are a total of N injection and production wells in the aquifer, then by the principle of superposition the potential at any point is given by[14]:

$$\Phi(x,y) = -\frac{\mu}{4\pi kh}\sum_{i=1}^{N}\{q_i \ln[(x-x_i)^2+(y-y_i)^2]\}+C_2 \quad (3.2)$$

It is assumed that the viscosities of the injected and native fluids are the same.

If the aquifer has gross fluid movement (flux) in the decreasing y direction, and there are no wells operating in the system, the potential at any point is given by[12]:

$$\Phi_e(x,y) = My_B + C_o \quad (3.3)$$

M = potentiometric gradient with respect to y, atmosphere per centimeter

C_o = constant potential along x-axis, atmospheres

The potential at any point due to the combined effects of N wells operating in an aquifer in which there is also a flux in the negative "y" direction is given by:

$$\Phi(x,y) = \frac{\mu}{4\pi kh}\sum_{i=1}^{N}\{q_i \ln[(x-x_i)^2+(y-y_i)^2]\}+\Phi_e(x,y)+C_2 \quad (3.4)$$

If a bubble of fresh water of radius r is injected into a saline aquifer which has flux in the negative y direction (see Figure 3.1), then there is a velocity component acting at every point on and inside the bubble in the negative y direction. The bubble will therefore flow in the direction of flux and after some time will have moved away from the injection well (see Figure 3.2).

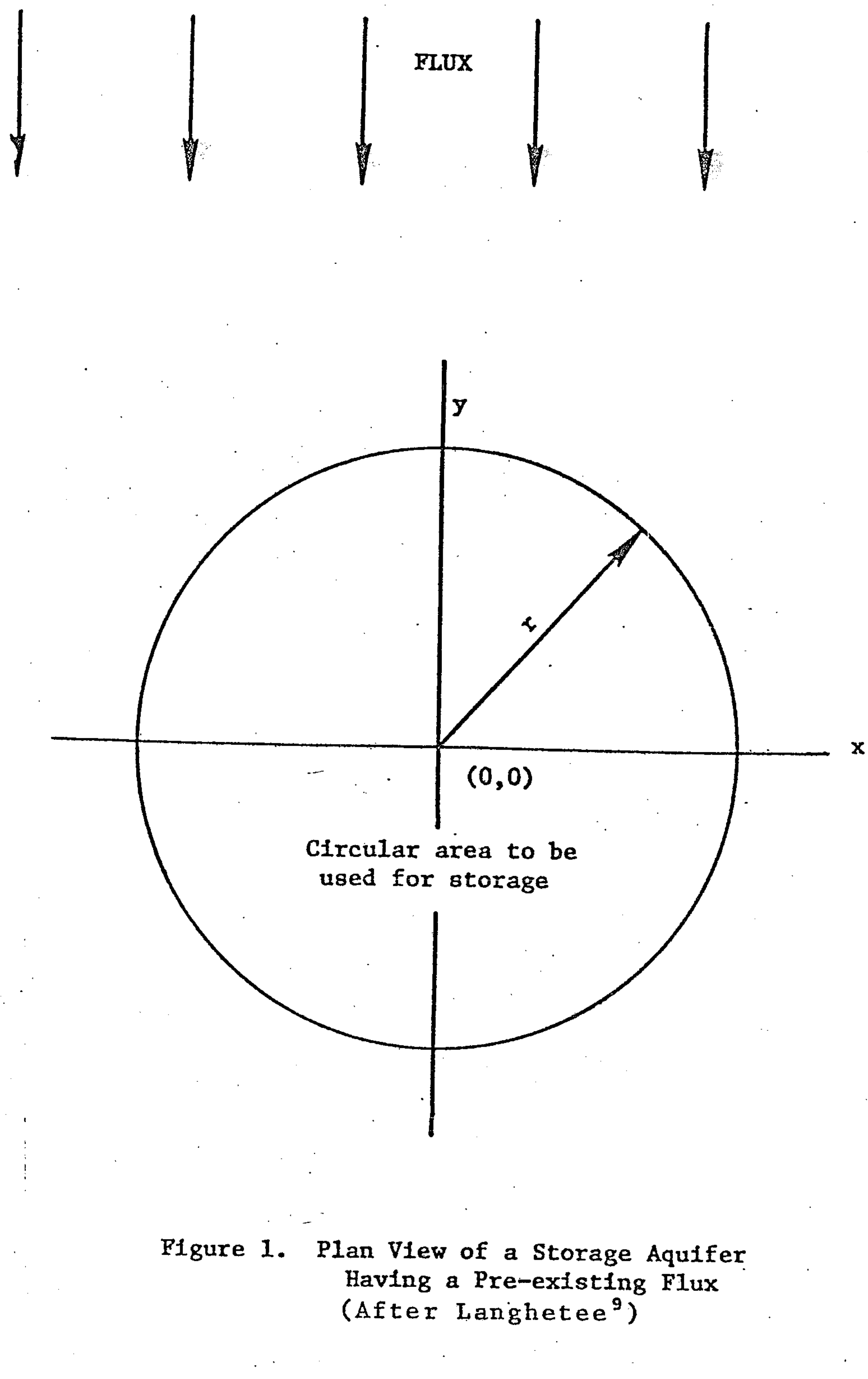


Figure 1. Plan View of a Storage Aquifer Having a Pre-existing Flux (After Langhetee[9])

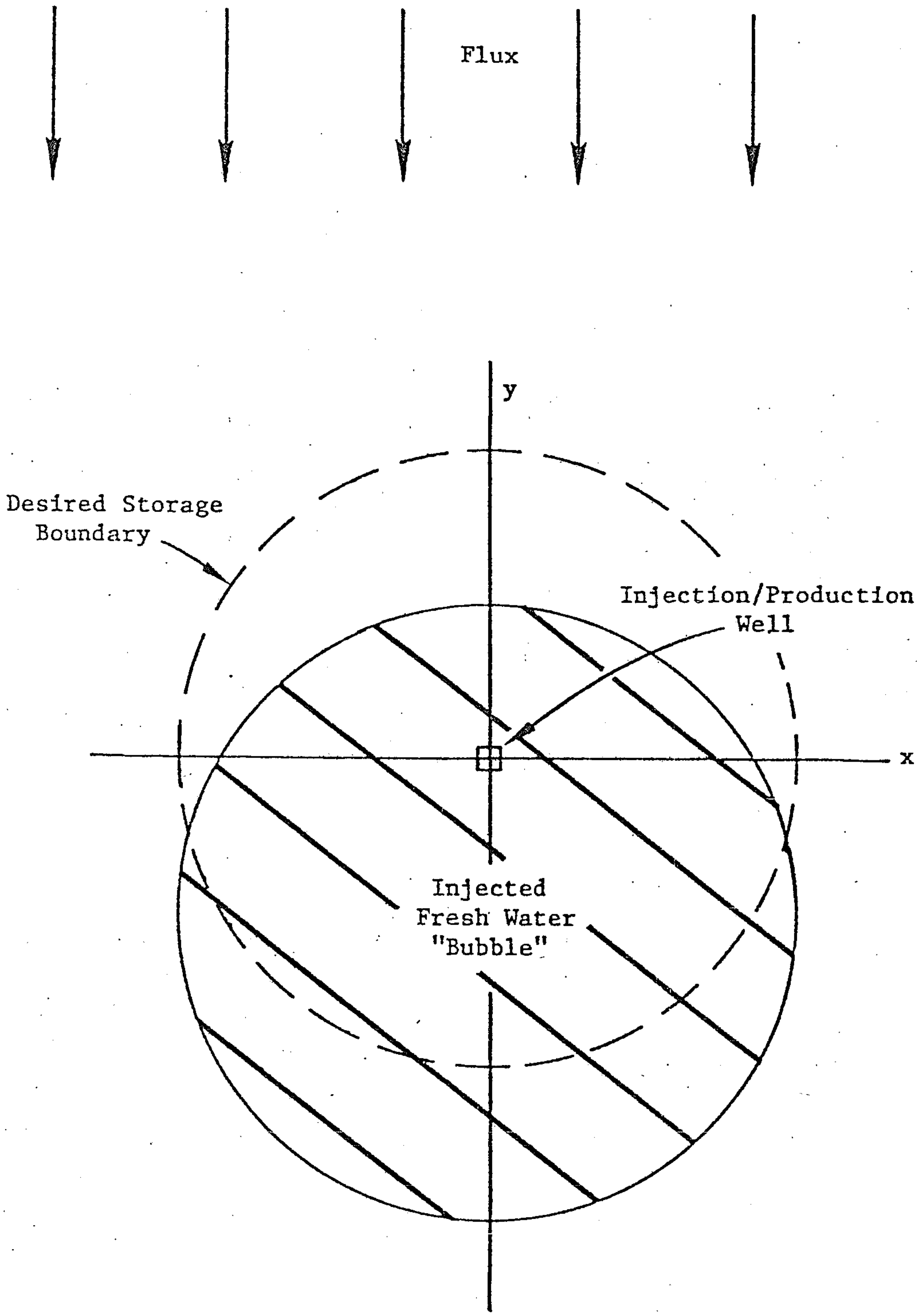


Figure 2. Migration of Injected Fresh Water "Bubble" as a Result of Flux (After Langhetee[9])

During production the upstream edge will breakthrough before the downstream edge.

If wells are positioned around the outside of the desired storage boundary (see Figure 3.3) and operated at flow rates so as to produce a velocity component of equal magnitude but opposite direction from the one caused by flux at every point on the boundary, then the enclosed area will become an isopotential. If there are no wells operating inside the boundary then there can be no flow across the boundary. If an injection well is operated inside the boundary, all fluid injected will move out radially until the boundary is reached.

In order to compute the rates at which the bounding wells should be operated, a desired potentiometric level for all points along the boundary and within the zone of stagnation must be selected. Let the potentiometric level selected be denoted by P_B, hence

$$\Phi(x_B, y_B) = P_B \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.5)$$

Thus, substituting into Equation 5

$$\Phi(x_B, y_B) = P_B = -\frac{\mu}{4\pi kh} \sum_{i=1}^{N} \{q_i . \ln[(x_B - x_i)^2 + (y_B - y_i)^2]\} + C_2 + My_B + C_o \quad \ldots\ldots \quad (3.6)$$

where N = number of bounding wells.

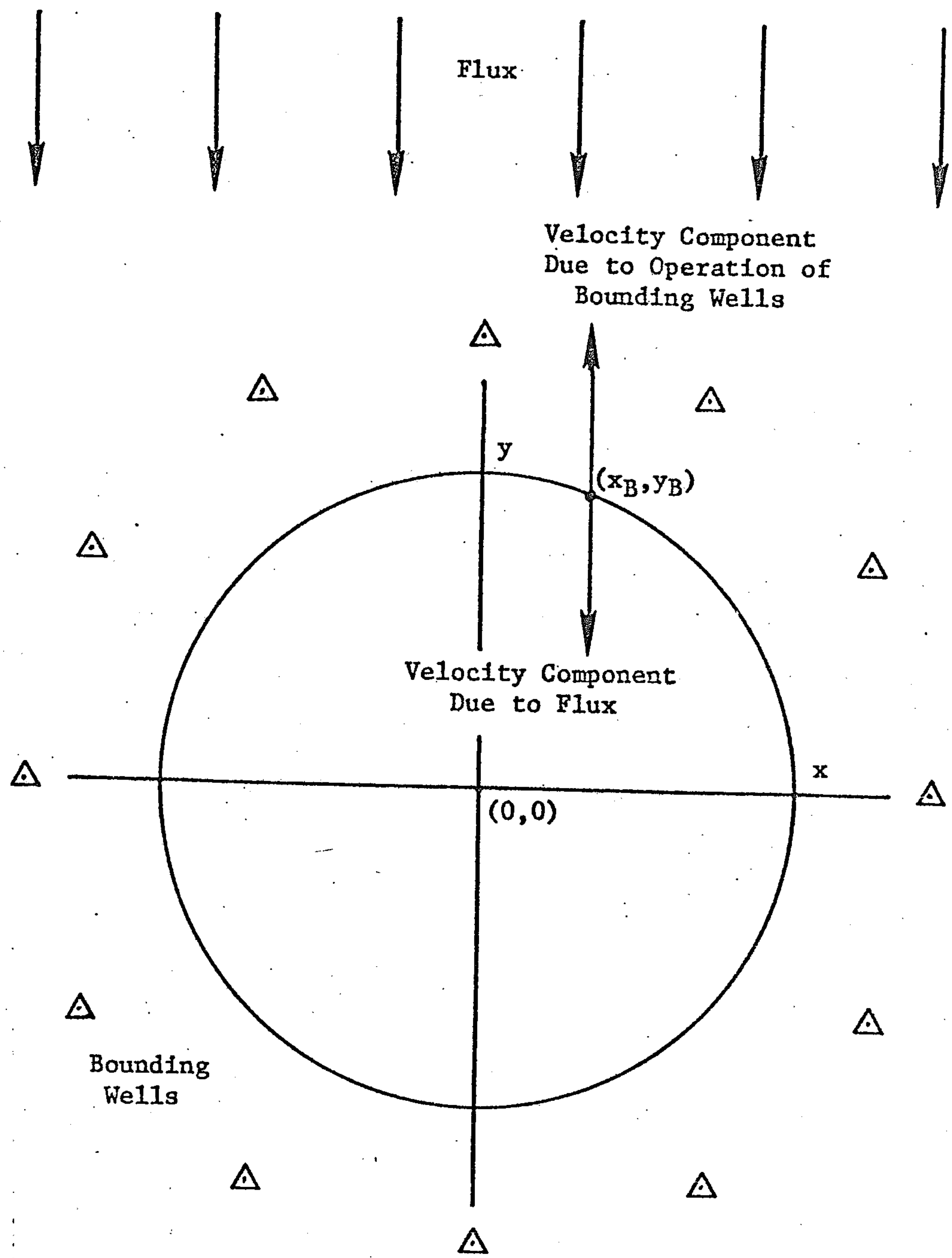


Figure 3. Arrangement of Bounding Wells to Negate the Effects of Flux (After Langhetee[9])

Rearranging Equation 3.6 we have

$$\sum_{i=1}^{N} q_i . \ln[(x_B - x_i)^2 + (y_B - y_i)^2] = \frac{4\pi kh}{\mu}$$

$$[My_B + (C_2 + C_o) - P_B] . . . . (3.7)$$

If we choose the value of P_B such that $(C_2 + C_o) - P_B = 0$, then Equation 3.7 simplified to:

$$\sum_{i=1}^{N} q_i . \ln[x_B - x_i)^2] = \frac{4\pi kh}{\mu} (My_B) . . . . (3.8)$$

This choice of P_B and the subsequent solution of Equation 3.8 for the values of q_i yields a balanced system. That is, the sum of the production rates is equal to the sum of the injection rates. The method of solution of Equation 3.8 for the bounding well flow rates is discussed later.

3.2 Dipping System

The effect of dip on fluids of different densities is that there is a tendency for the less dense fluid to migrate in the updip direction. The updip and downdip positions of an injected fluid have been evaluated by Painter for a linear system dipping in the x direction as:

$$x_\mu = \frac{q_i t}{2A\phi} + 4.94 \times 10^{-7} \frac{k\Delta\rho g.\sin.\alpha.q_i t^2}{\mu L A \phi^2} . . . . . (3.9)$$

$$x_d = \frac{q_i t}{2A\phi} - 4.94 \times 10^{-7} \frac{k\Delta\rho g.\sin.\alpha.q_i t^2}{\mu L A \phi^2} . . . . . (3.10)$$

where

x_μ = updip frontal location at t, cm

x_d = downdip frontal location at t, cm

q_i = injection rate, cc/sec

A = total cross-sectional area, cm^2

L = total length, cm

t = time, sec

ϕ = porosity

g = gravitational acceleration, cm/sec^2

k = permeability, darcies

$\Delta\rho$ = density difference, gm/cc

α = dip angle, degrees

μ = viscosity, cp

Assumptions made by Painter[5] are:

(1) Piston like displacement

(2) Fluids of small and constant compressibility

(3) Homogeneous and isotropic medium of finite length, L

(4) Breakthrough has not occurred.

In the linear system just described, the displaced fluid always moves linearly towards the ends of the system.

The first derivatives of Equation 3.11 and 3.13 yield the velocities at which the respective fronts will move.

$$\frac{dx_\mu}{dt} = V_\mu = \frac{q_i}{2A\phi} + 9.88 \times 10^{-7} \frac{k\Delta\rho g.\sin.\alpha q_i t}{\mu L A\phi^2} \quad . \; . \; . \; (3.11)$$

$$\frac{dx_d}{dt} = V_d = \frac{q_i}{2A\phi} - 9.88 \times 10^{-7} \frac{k\Delta\rho g.\sin.\alpha q_i t}{\mu L A\phi^2} \quad . . .(3.12)$$

The first term is the velocity at which the fronts would move if there was no density difference. The second term is due to gravity and the density difference and can be rewritten as:

$$V_g = 9.88 \times 10^{-7} \frac{k\Delta\rho g.\sin.\alpha}{\mu\phi} \left[\frac{q_i t}{AL\phi}\right]^n \quad . . . .(3.13)$$

where

V_g = velocity due to dip

n = 1

The bracketed term is a dimensionless quantity relating volumes injected at any time to total pore volume of the system.

The radial system is much more difficult to solve because the streamlines are constantly changing in strength and direction. There must also be movement of displaced fluid around the mass of injected fluid. Painter undertook an experimental study to evaluate these effects. He found that in modifying Equation 3.13 to describe a radial system, n would not be unity, due to the circulation around the edge of the injected bubble. Therefore, in a radial system the gravity component of fluid velocity is given by:

$$V_g = 9.88 \times 10^{-7} \frac{k\Delta\rho g \sin \alpha}{\mu\phi} \left[\frac{q_i t}{\pi r_e^2 h\phi} \right]^n \quad . \; . \; . \; . \; . \quad (3.14)$$

where

r_e = radius related to effective pore volume of system

In equation 3.14 r_e is defined by Painter[5] as the distance to a circular boundary of constant potential outside the storage area. Tate[10] considered r_e to be that radius from the center of the stored bubble beyond which the circulation of the native fluid around the bubble becomes negligible. The present study considers r_e to be the same as the desired storage radius.

Painter found that the exponent in Equation 3.14 had a value of 0.17. This value was based on experimental runs in a thin radial miniaquifer which was treated as a two-dimensional system with a dip of seven and one half degrees. D'Amico, working on a different miniaquifer, found values for n which varied from .018 to .205 for angles of five to twenty degrees. Since this study used the same miniaquifer as Painter, a value of .17 was used.

3.3 Negating Gravity Effects in Dipping Systems

The present study envisions that a system of bounding wells can be used to negate, or at least minimize the harmful effects of dip. This requires that the bounding wells be operated in such a manner that the stored fresh water bubble is never allowed to migrate relative to the injection production well. This necessitates that the injected bubble remain circular and concentric with the well throughout the injection, storage, production (ISP) cycle.

Referring to Figure 3.4(a) and considering the potential at points A and B on the periphery of the injected bubble of more dense fluid than the native fluid, it is seen that the potential at A is greater than at point B. The potential distribution between the two points is taken to be linear as shown in (b). Thus the potential gradient along the line connecting points A and B is

$$\frac{\Phi(x_A, y_A) - \Phi(x_B, y_B)}{y_A - y_B} = \frac{\Delta\Phi_{\overline{AB}}}{\Delta y_{\overline{AB}}} \quad \ldots\ldots\ldots\ldots (3.15)$$

The potential gradient tending to cause flow along the line connecting points A and B can be related to a velocity through Darcy's Law. Thus,

$$V_g = -\frac{K}{\mu\phi_D}\frac{\Delta\Phi_{\overline{AB}}}{\Delta y_{\overline{AB}}} \quad \ldots\ldots\ldots\ldots (3.16)$$

If bounding wells can be operated such that they produce a potential gradient equal in magnitude but opposite in sign to that of Equation 3.16, there will be no tendency for flow along a line connecting points A and B. The same argument can be applied to all pairs of points on a line parallel to the y axis. For the bubble to remain motionless

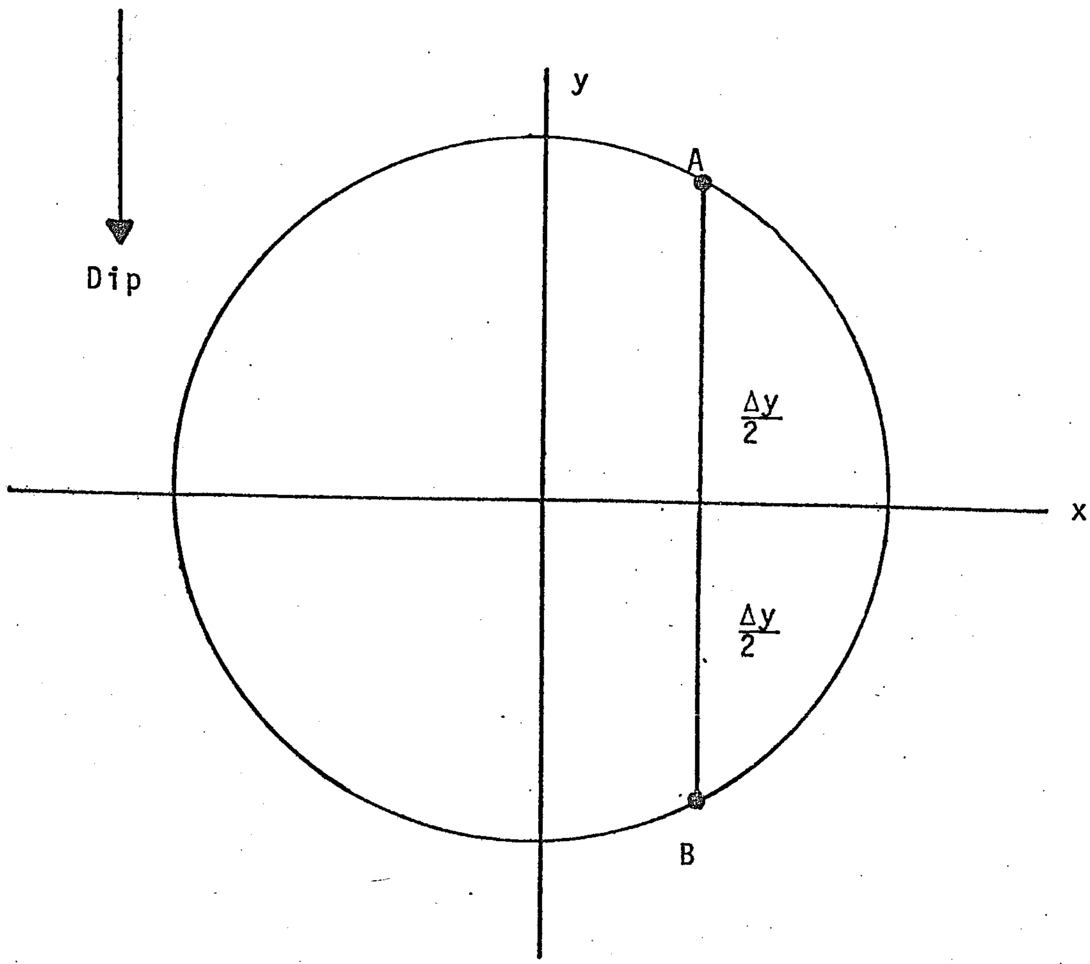


Figure 3.4(a) - Plain View of a More Dense Fluid Bubble Injected Into a Dipping Aquifer

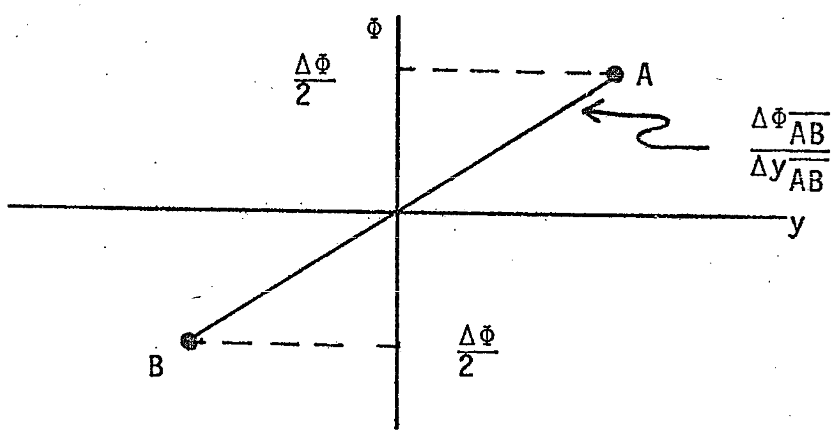


Figure 3.4(b) - Potential Distribution of More Dense Fluid Bubble in a Dipping Aquifer

bounding well flow rates must be selected which satisfy these requirements for all pairs of points simultaneously. These rates are dependent on bubble size and continually change throughout injection and production.

Substituting the expression for velocity due to gravity derived in Equation 3.14 into Equation 3.16 allows us to solve for the potential gradient due to gravity from point A to B.

$$\frac{\Delta\Phi_{\overline{AB}}}{\Delta y_{\overline{AB}}} = -\ 9.88 \times 10^{-7}\ \Delta\rho\ g\ \sin\alpha\ \left[\frac{q_i t}{\pi r_e{}^2 h\phi}\right]^n \quad . \ . \ . \ . (3.17)$$

Thus the potential due to gravitational effects at any point such as A can be expressed as:

$$\Phi(x_A, y_A) = \frac{\Delta\Phi_{\overline{AB}}}{\Delta y_{\overline{AB}}}\ y_A + C_o \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (3.18)$$

where

C_o = constant potentiometric level along the x axis (atmospheres)

If N bounding wells are operating in the dipping system then using superpositions and Equations 3.2 and 3.17 the total potential at a point on the periphry is

$$\Phi(x_A, y_A) = -\ \frac{\mu}{4\pi kh} \sum_{i=1}^{N} \{q_i . \ln[(x_A - x_i)^2 + (y_A - y_i)^2]\}$$

$$+\ C_2 + \frac{\Delta\Phi_{\overline{AB}}}{\Delta y_{\overline{AB}}}\ y_A + C_o . \ . \ . \ . \ . \ . \ . \ . \ . \ (3.19)$$

rearranging

$$\sum_{i=1}^{N} \{q.\ln [(x_A-x_i)^2 + (y_A-y_i)^2]\} = \frac{4\pi kh}{\mu} [\frac{\Delta\Phi_{\overline{AB}}}{\Delta y_{\overline{AB}}} y_A$$

$$+ (C_o + C_2) - \Phi(x_A-y_A)]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.20)$$

In order for the bubble to remain motionless every point on the periphry of the bubble must be at the same potential. If we select this value of potential such that $(C_2 + C_o) - \Phi(x_A,y_A) = 0$ then Equation 3.20 simplifies to

$$\sum_{i=1}^{N} q_i.\ln[(x_A-x_i)^2+(y_A-y_i)^2]=\frac{4\pi kh}{\mu} \frac{\Delta\Phi_{\overline{AB}}}{\Delta y_{\overline{AB}}} y_A \quad . \quad .(3.21)$$

The solution of Equation 3.21 for the values of q_i yields a balanced system.

3.4 Solution Technique

With the selection of N different and distinct points along the boundary, N equations in N unknowns (the value of q_i) can be set up using Equation 3.21. These equations can then be solved simultaneously for the flow rates of the bounding wells. Lin[19] found that, by taking only N points and generating N equations, in some cases the generated matrix was, in mathematical terms, ill-conditioned. That is, it was difficult to obtain a solution vector using techniques such as Gauss elimination, Crout reduction, etc. He found that by taking M boundary points

($M \geq 2N$) and generating M equations in N unknowns and finding the best solution vector, in the least-squares sense, to this set of equations, the problem of the ill-conditioned matrix was overcome.

The method for solving M equations in N unknowns ($M > N$) to find the best solution vector in the least-squares sense can be found in any good numerical methods text (Carnahan et al., 1969). Briefly, the method is this: Suppose we have a set of linear equations such that

$$AQ = B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.22)$$

where A is an M by N matrix, Q is an N by 1 matrix, and B is an M by 1 matrix. If each side of Equation 3.22 is pre-multiplied by the transpose, A^T (an N by M matrix), of A

$$(A^T A)Q = (A^T B) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.23)$$

then the M equations in N unknowns are reduced to N equations in N unknowns, the solution vector of which is the best in the least-squares sense.

A computer program has been written to perform all the calculations mentioned in this section. A description and listing of the program, in FORTRAN IV language for use on an IBM 360/65 system, is given in the Appendix B.

It should be noted that the above procedures are applicable only to systems of infinite areal extent. Therefore in order to experimentally verify the computational techniques it was necessary to employ the well known method of images to account for boundaries that are obviously present in a laboratory size system.

CHAPTER IV

PROCEDURE

All experiments were performed on the two dimensional radial mini-aquifer constructed by Painter and shown schematically in Figure 4.1. A detailed description of the construction technique has been reported by Painter[5]. Table 4.1 details the physical parameters of the mini-aquifer. The mini-aquifer represents half of a radial system bounded by two no-flow boundaries and isopotentials at each end as used by Esmail[6] (see Figure 4.2). The center well is surrounded by bounding wells. These are used to negate flux in horizontal aquifers, and effects of gravity in dipping aquifers.

Table 4.1 - Physical Characterists of the Mini-Aquifer

k = 5.09 darcies	length = 230
h = 1.10 cm	width = 110 cm
ϕ = .25	r_w = .625

All experimental runs were made with the system initially saturated with a mixture of 45 percent soltrol and 55 percent naptha by volume. For the horizontal runs, the injected fluid was the same as the native fluid except

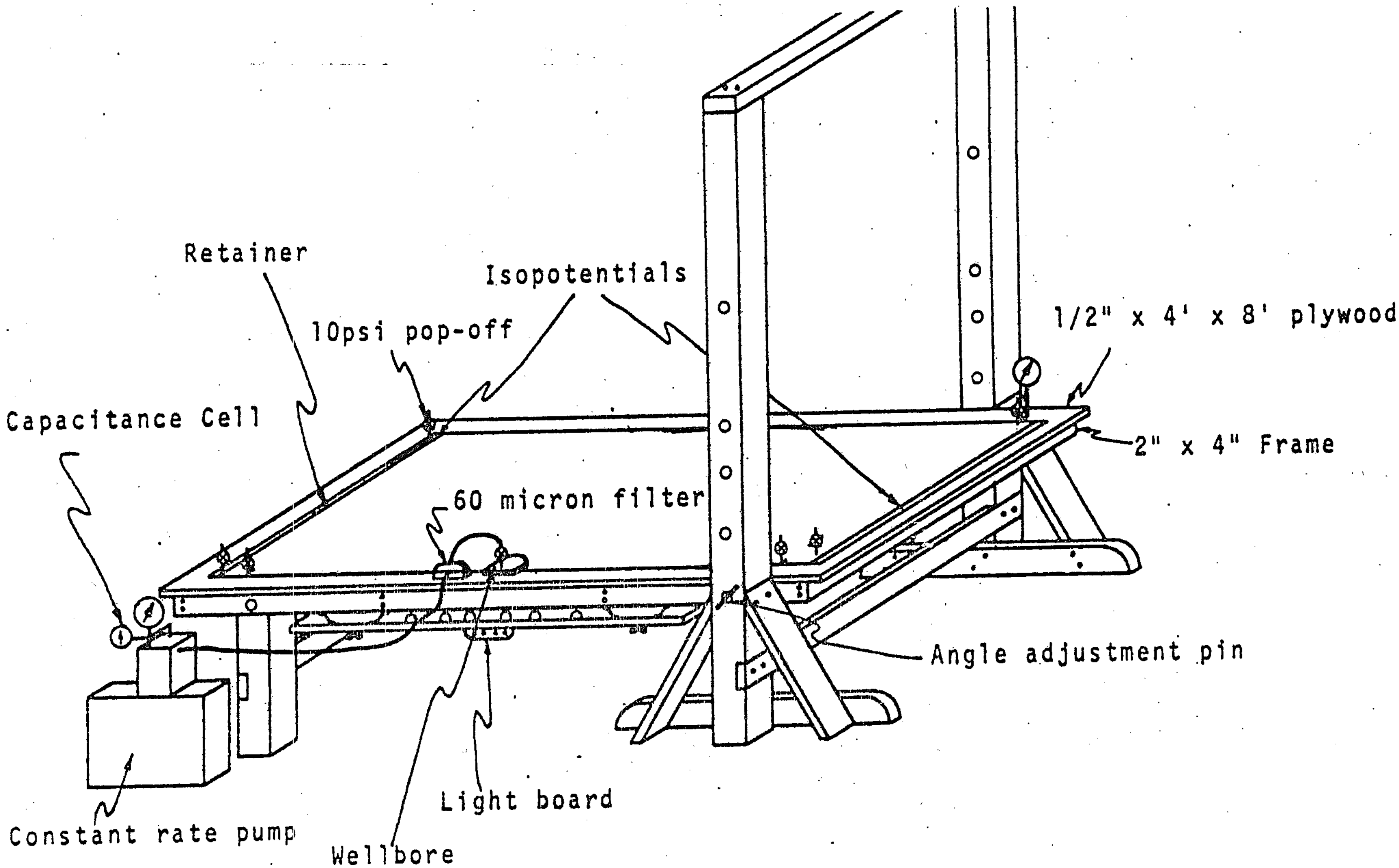


FIGURE 4-1 SCHEMATIC DIAGRAM OF THE MINI-AQUIFER AND SUPPORTING STRUCTURE. (AFTER PAINTER)

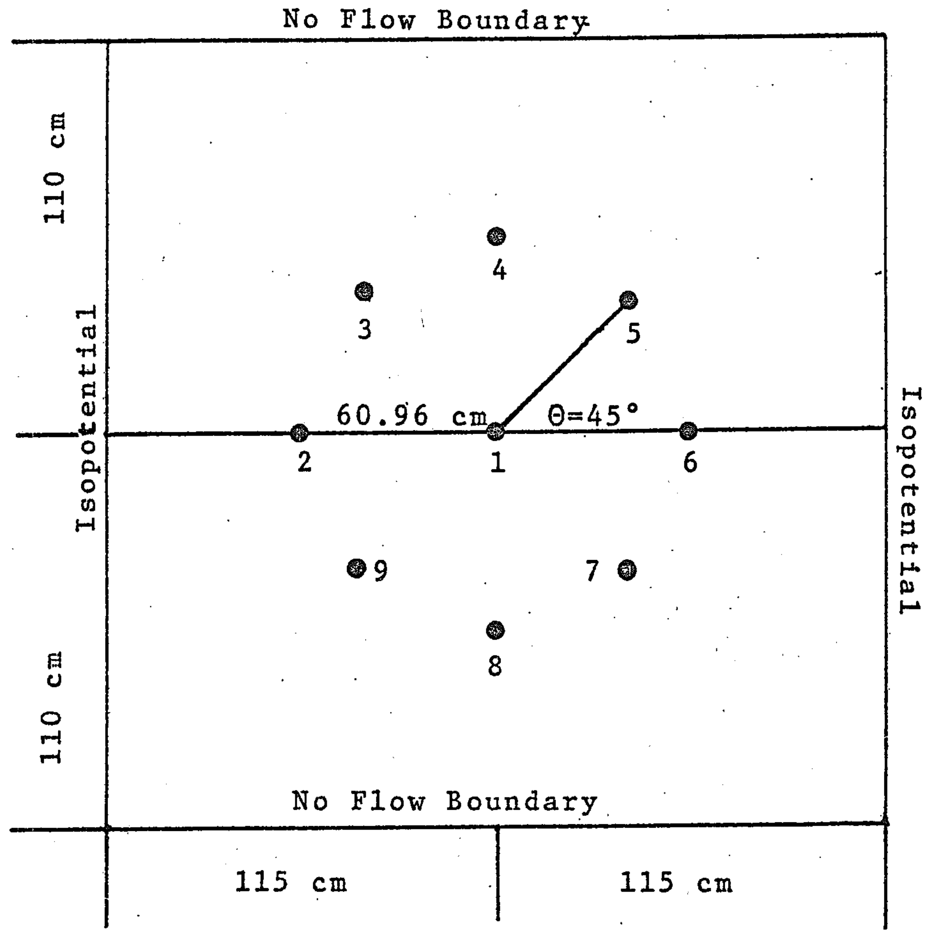


Figure 4.2 - Schematic of the System Represented by the Mini-Aquifer

for the addition of a soluble dye, which allows the injected fluid front to be seen, but does not effect the properties of the injected fluid. For runs in which the aquifer was dipping, carbontetrachloride was added to the injected fluid to increase its density. The soltrol-naptha mixture has the same viscosity of carbontetrachloride. Thus when carbontetrachloride is added the mobility ratio between injected and native fluid remains one. For runs in which the recovery efficiency was desired, 2.5 percent by volume iodobenzene was added to alter the dielectric properties of the injected fluid. By use of a Sargent Model V Chemical Oscellometer the recovery efficiency was found. The oscillometer detects a change in capacitance, which indicates that native fluid is being produced. Table 4.2 list the properties of all fluids used in this investigation.

Langhetee's[9] computer model which predicts the flow rates at which the bounding wells must be operated at in order to negate flux in an infinite horizontal aquifer was modified[13] to take into account the physical boundaries of the miniaquifer used in this investigation. Superposition and imaging[14] were used to create the isopotentials and the no flow boundaries. The model was also modified to solve for bounding well flow rates operating in a dipping system in which the injected fluid and native fluid are of different densities. The complete listing of the computer model can be found in Appendix B.

Fluid	Density at 22°C (gm/cc)	Viscosity at 22°C (cp)
[1]Naptha	0.747	0.570
[2]Soltrol 170	0.771	2.504
[3]Carbon tetrachloride	1.590	0.992
[4]Iodobenzene	1.832	1.573

[1]Naptha; V.M. & P.; Humble Oil and Refining Co.; Baton Rouge, La.

[2]Soltrol 170; Aliphatic Hydrocarbon; Phillips Petroleum Co.; Bartlesville,OK.

[3]Carbon Tetrachloride; Technical Grade; F.H. Ross & Co.; Baton Rouge, La.

[4]Iodobenzene; Aldrich Chemical Co., Inc.; Milwaukee, Wisconsin

Table 4.2 - Properties of Pure Fluids Used in Experimental Runs (From Tate[10])

All wells were operated by constant rate pumps. The leading edges of the injected bubble were traced at 600 second intervals during injection and production. The tracings can be seen in Appendix C. After each run, the injected fluid was completely flushed from the system.

For all experimental runs involving flux, the flux was created by a static head connected into an isopotential at one end of the miniaquifer and recovered from an isopotential at the other end. The potential gradient due to flux can be changed by increasing or decreasing the static head. In runs involving dip, the potential gradient was caused by dip and density difference and could be changed by altering one or both of these parameters. For convenience, the injected fluid was always the more dense fluid so that the tendency to flow was always in the down dip direction, considered here to be the negative y direction.

A run consisted of injecting a bubble of fluid out to a desired storage radius and then producing it through the same well used for injection. When recovery efficiency was not required, the injected fluid was produced until the produced fluid was 100 percent native fluid. In the horizontal runs with a constant flux the rates of the bounding wells were also constant. In the dipping system, the rates of the bounding wells change with the radius of the bubble of injected fluid, thus necessitating gear changes on the constant rate pumps throughout the run.

In runs where recovery efficiencies were required, breakthrough was considered to have occurred when the concentration of reservoir fluid in the produced stream reached five percent.

Using the technique described above the following experiments were performed:

(1) Horizontal, no flux, bounding wells not operating

(2) Horizontal with flux, bounding wells not operating

(3) Horizontal with flux, bounding wells operating

(4) Dipping, bounding wells not operating

(5) Dipping, bounding wells operating

The purpose of the horizontal runs was to test the validity of Langhetee's computer model listed in Appendix B.

CHAPTER V

ANALYSIS OF EXPERIMENTAL RESULTS

5.1 General

The process of confining a bubble of stored injected fluid by use of bounding wells was studied by means of a series of runs in the mini-aquifer (Table 5.1). The accomplishments of this study were threefold:

(1) The theoretical computational procedure developed by Whitehead and Langhetee[12] was verified experimentally.

(2) A computational procedure for predicting bounding well rates to negate the effects of dip was developed.

(3) This procedure was experimentally verified.

5.2 Horizontal Results

The experimental work on the horizontal system was designed to test Langhetee's mathematical solution. Figure 5.1 is a graph of well flow rate versus pre-existing gradient, calculated from the computer program listed in Appendix B. For the experiments a gradient was selected such that the bounding well flow rates needed to offset the chosen gradient could be attained by the constant rate pumps.

TABLE 5.1

Run No	Dip Angle Degrees	Density Difference gm/cc	Injection & Production Rates cc/min	Gradient Ft/Mile	Storage Time (sec.)	Bounding Wells in Operation?
1	0	0	6	0	0	No
2	0	0	6	2000	0	No
3	0	0	6	2000	0	Yes
4	15	.3	8	0	0	No
5	15	.3	8	0	0	Yes
6	15	.4	8	0	0	No
7	15	.4	8	0	0	Yes
8	30	.4	8	0	0	No
9	30	.4	8	0	0	Yes
10	30	.55	8	0	0	No*
11	30	.55	8	0	0	Yes*
12	30	.55	8	0	5400	Yes*
13	30	.55	8	0	5400	Yes*
14	0	.55	8	0	5400	No*

*Runs in which the recovery efficiency was found.

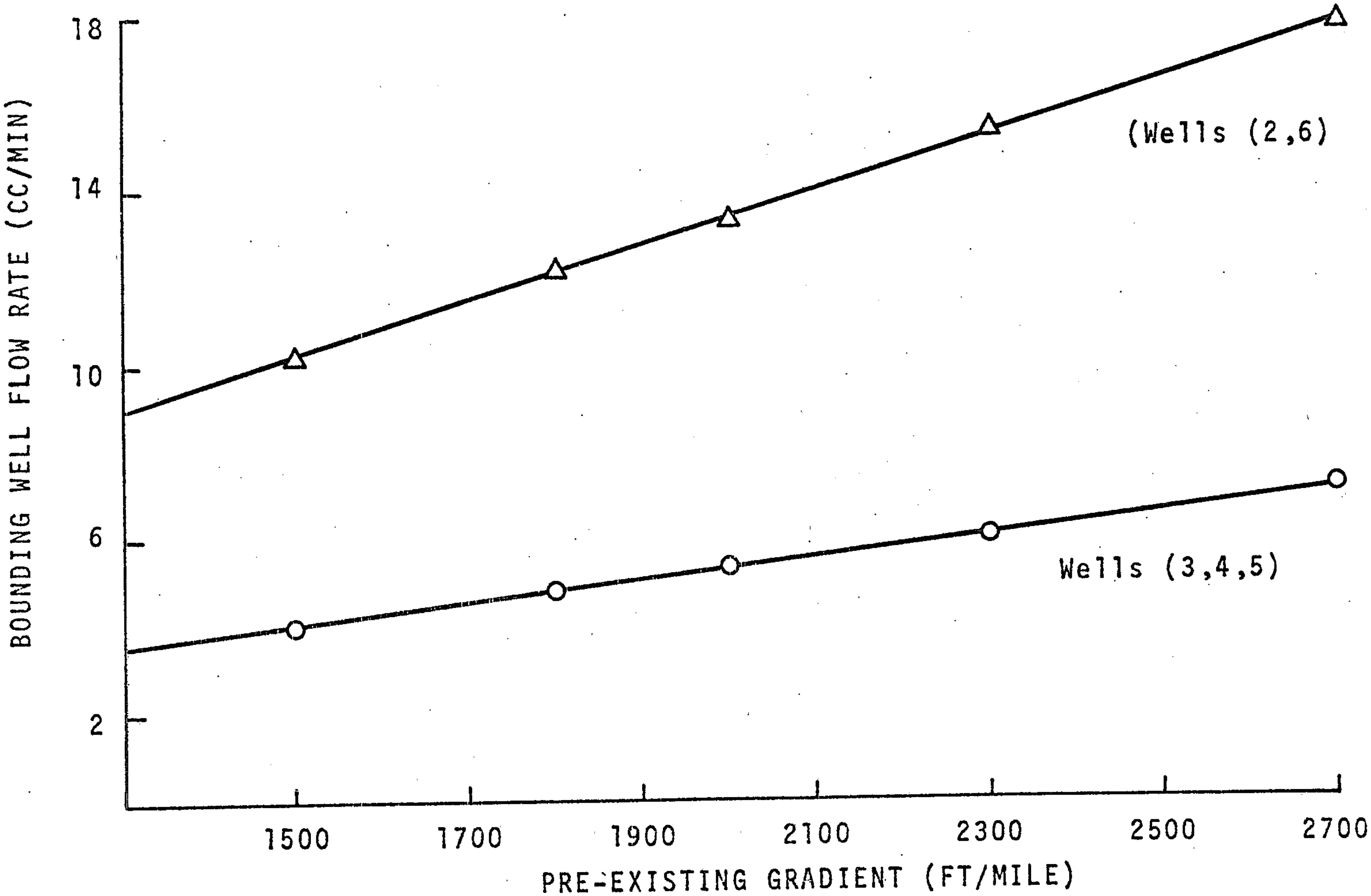


FIGURE 5.1 - The Bounding Well Flow Rates Needed at Specified Wells to Neutralize the Indicated Potentiometric Gradient

Figure 5.2 is a tracing of the front of the injected fluid at various times during the injection cycle of run one. The flow is essentially radial. By comparison, the frontal configuration during the injection cycle of run two (Figure 5.3) is far from being radial. This distortion is caused by the addition of a velocity component due to flux. The velocity of the flux adds vectorially to the velocity resulting from injection into the well.

The production phases of runs one and two are shown in Figures 5.4 and 5.5, respectively. Breakthrough occurs in Figure 5.5 long before it does in Figure 5.4. Figures 5.6 and 5.7 show frontal positions during the injection and production phases of run three. Here it can be seen that the effects due to flux have essentially been negated. Langhetee[9] pointed out that in a symetrical arrangement of bounding wells the wells opposite each other in respect to the x-axis, assuming flux in the negative y direction, will have equal but opposite flow rates. Also for bounding wells on the x-axis, the flow rates will essentially be zero. Therefore, referring back to Figure 4.2, wells four and eight had flow rates of zero thereby reducing the number of bounding wells to six. By comparing runs one and three, we see that a gradient of 2000 feet per mile has been negated and this gradient is much larger than one that might occur in nature.

5.3 Studies in a Dipping Mini-Aquifer

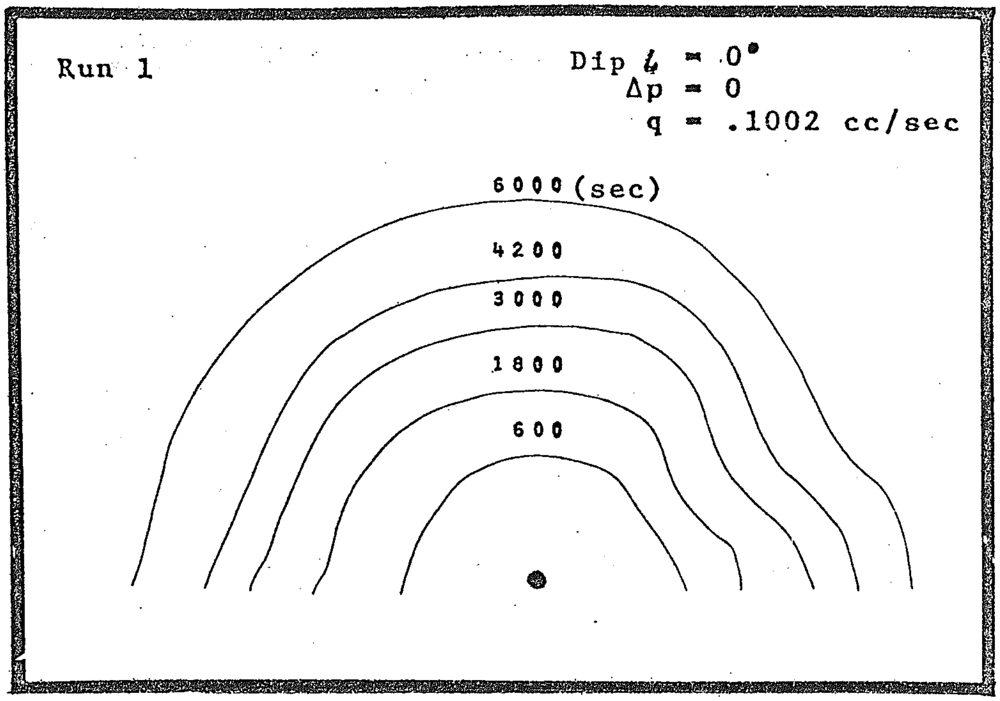


Figure 5.2 - Injection Phase of Run 1 (No-Pre-existing Gradient in Aquifer, Bounding wells Not Operating)

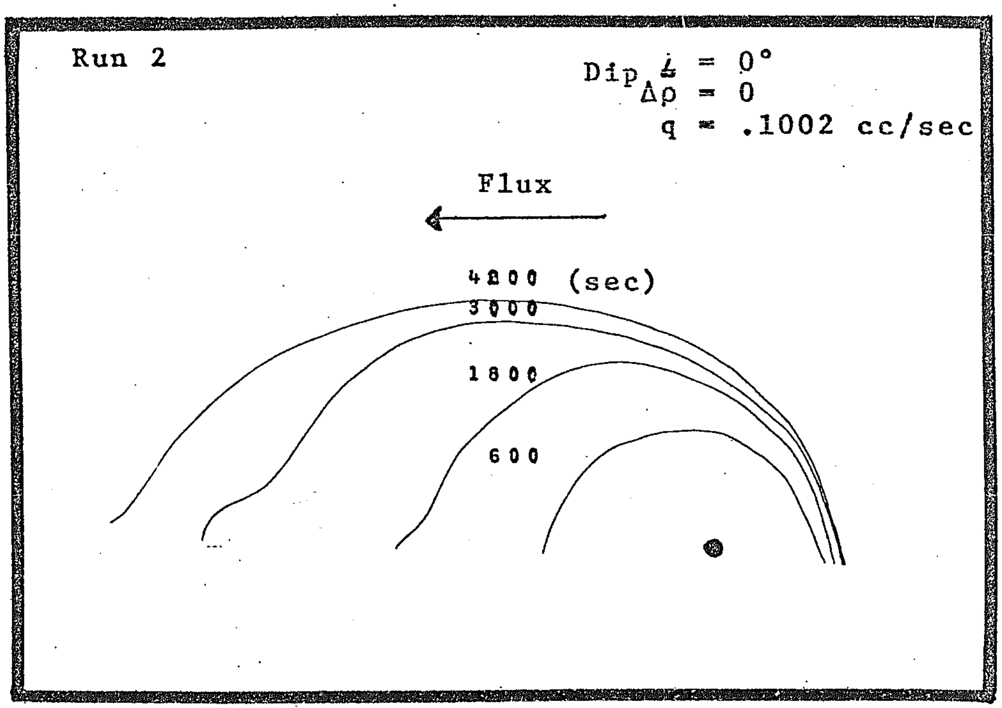


Figure 5.3 - Injection Phase of Run 2 (Pre-existing gradient 2000 ft/mile, bounding wells not operating)

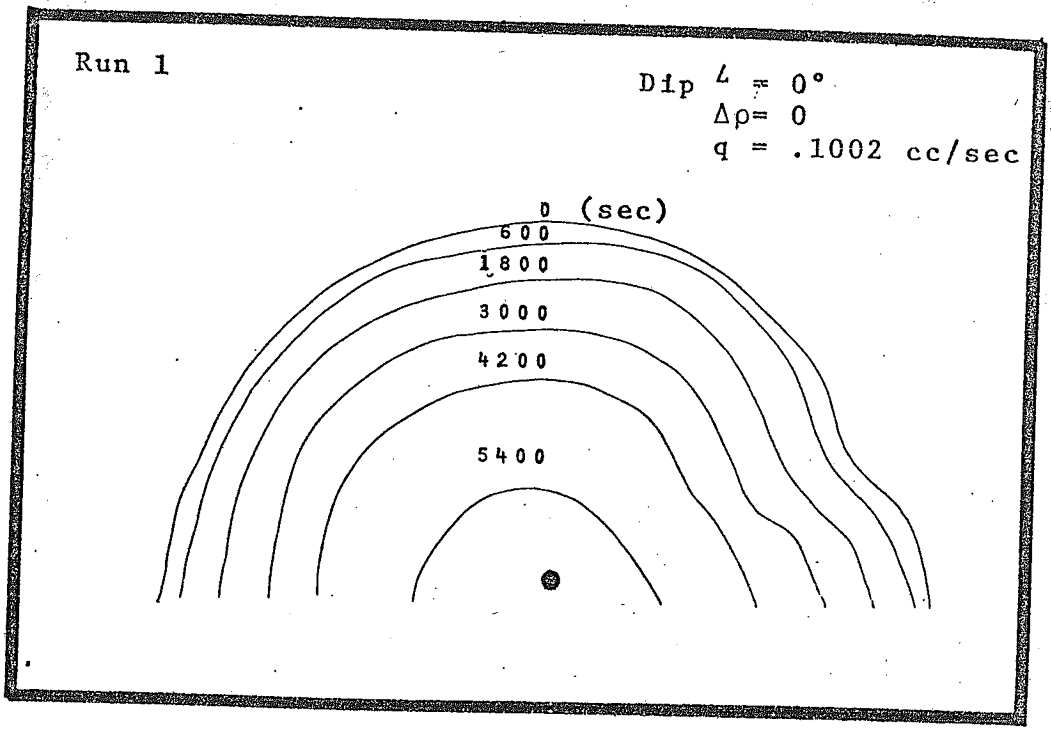


Figure 5.4 - Production Phase of Run 1 (No Pre-existing Gradient, Bounding Wells not Operating)

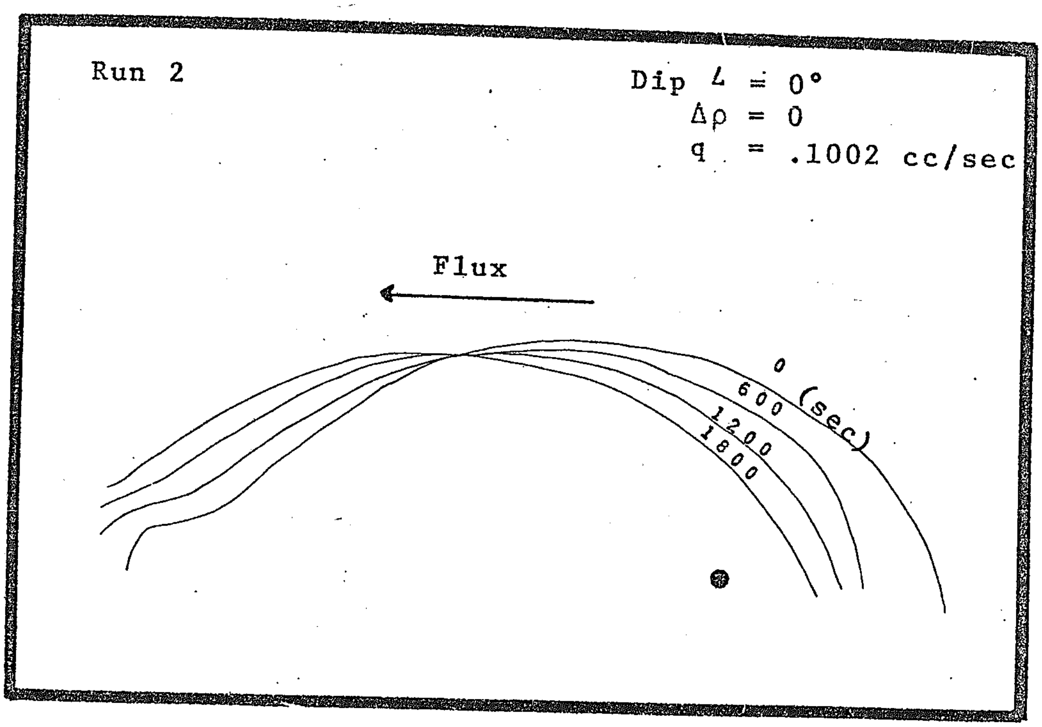


Figure 5.5 - Production Phase of Run 2 (Pre-existing Gradient 2000 ft/mile, Bounding Wells Not Operating)

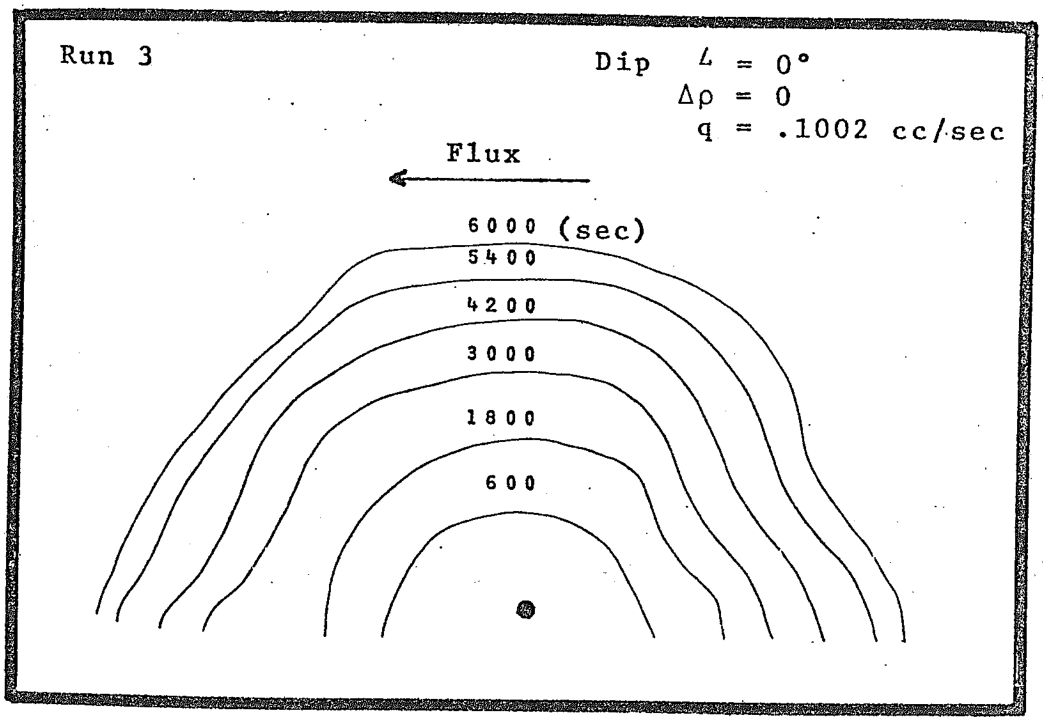


Figure 5.6 - Injection Phase of Run 3 (Pre-existing Gradient 2000 ft/mile, Bounding Wells Operating)

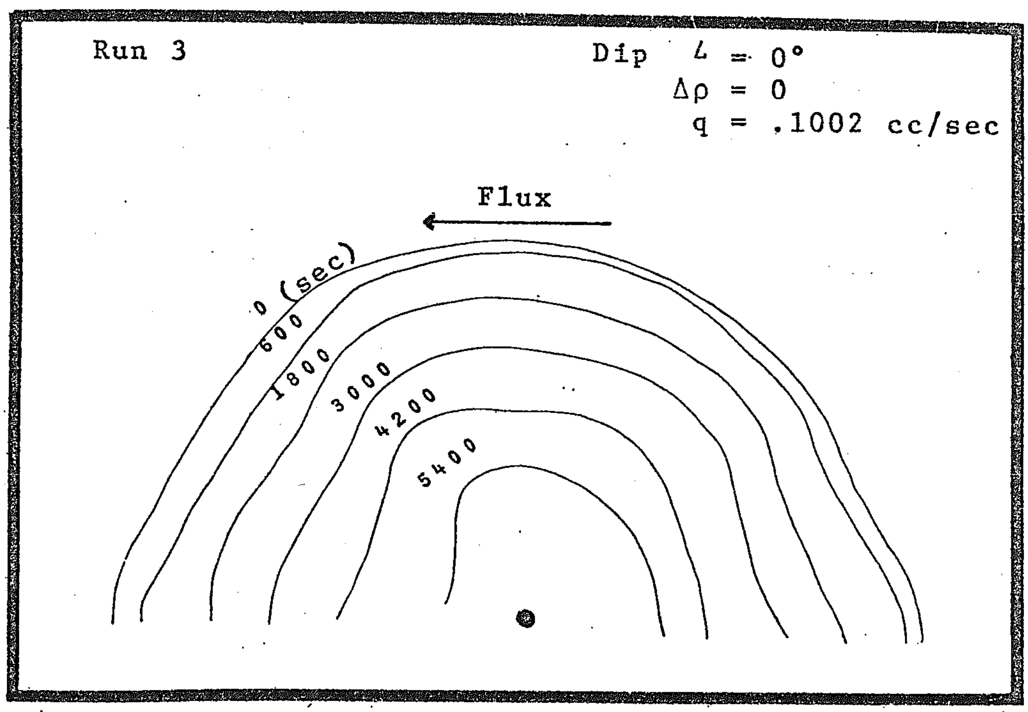


Figure 5.7 - Production of Run 3 (Pre-existing Gradient 2000 ft/mile, Bounding Well Operating)

The primary objective of this investigation was the study of the use of bounding wells in order to negate the effects of gravity when a fluid of different density from the native fluid is injected into a dipping aquifer.

As reported by Painter, the front of a bubble of a fluid of different density, injected at a constant rate, appears radial at first, but as injection continues the migration of the injected fluid increases (see Figure 5.8). If the injection fluid is the more dense fluid when the well is put on production, the updip front will breakthrough before the downdip front. In severe cases, the downdip front will not breakthrough, but will migrate away from the producing well. Premature breakthrough will, of course, cause the recovery efficiency to decrease. (See Figure 5.9).

Recall equation 3.13

$$V_g = 9.88 \times 10^{-7} \times \frac{k\Delta\rho g.\sin\alpha}{\mu\phi}\left[\frac{r^2}{r_e^{\,2}}\right]^{0.17}$$

it can be rewritten as:

$$V_g = 9.88 \times 14^{-7} \times \frac{k\Delta\rho g.\sin\alpha}{\mu\phi}\left[\frac{r^2}{r_e^{\,2}}\right]^{0.17} \quad . . . .5.1$$

where r is the radius of the injected bubble at any time,t.

Equation 5.1 shows that there are four parameters which effect velocity due to gravity. These are density difference, dip angle, radius of the injected bubble, and r_e.

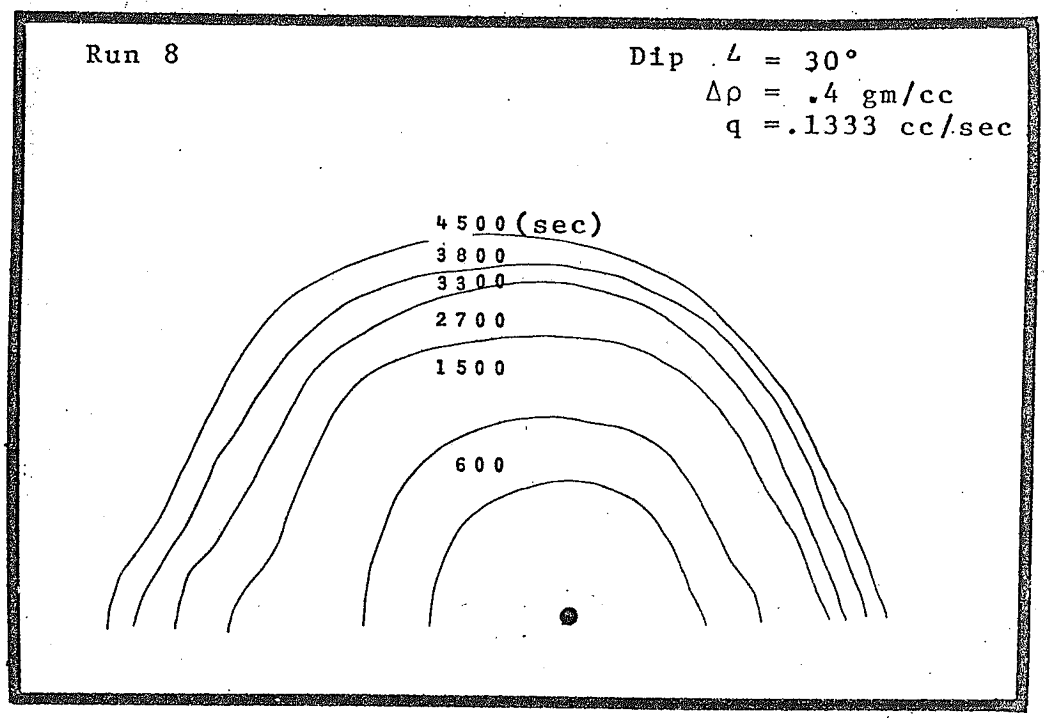


Figure 5.8 - Injection of a More Dense Bubble Into a Dipping Aquifer

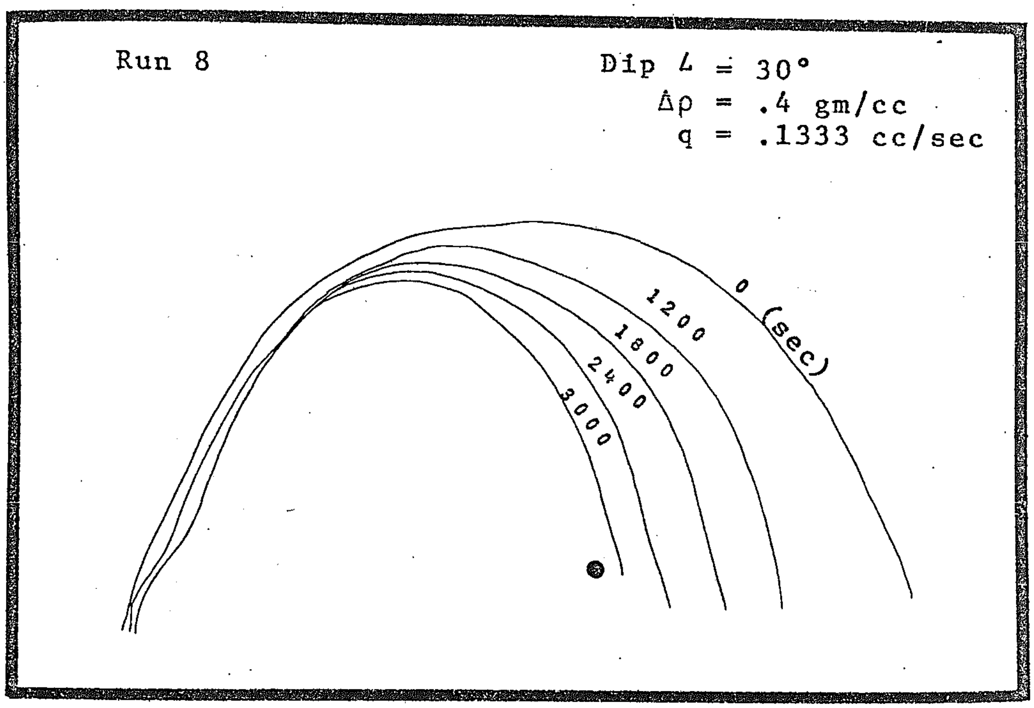


Figure 5.9 - Production of a More Dense Bubble from a Dipping Aquifer

In order to use equation 5.1, a satisfactory value for r_e must first be determined. This was done by calculating the velocities due to gravity for various r_e's. The bounding well flow rates necessary to offset these velocities were also computed. Experiments were performed on the mini-aquifer using the computed bounding well flow rates for the various r_e's. It was found that the value of r_e required to negate the effects of gravity in the dipping mini-aquifer was equal to the radius of the proposed storage area.

The velocity due to gravity is directly proportional to dip angle, density difference and radius of the injected bubble. Figure 5.10 is a plot of a typical bounding well flow rate versus radius of injected bubble for different dip angles. The bounding well flow rates required to offset gravitational effects increase with radius of the injected bubble and dip angle. Figure 5.11 is a plot of a typical bounding well flow rate versus radius of injected fluid bubble for fluids of different densities. Bounding well flow rates increase with increases in density difference and radius.

Figures 5.12 and 5.13 are a comparison between the injection phases of runs ten and eleven. The front in run eleven is more symmetrical than the front in run ten. The injection phase of run eleven is in fact very similar to that of the horizontal run with no flux. This comparison can be seen in Figures 5.14 and 5.15. Figures 5.16 and 5.17 are a comparison between the production phases of runs

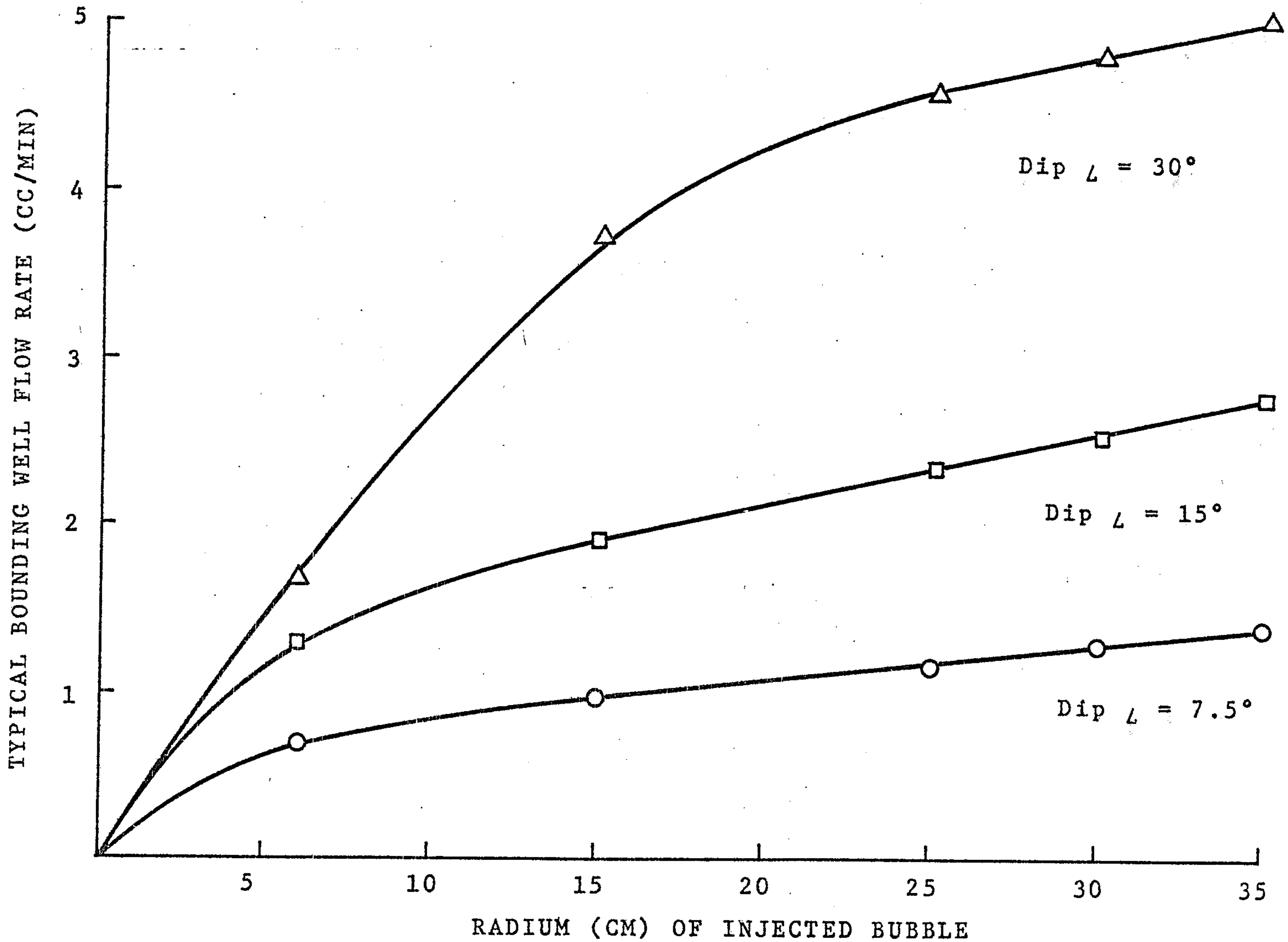


Figure 5.10 - Plot of Typical Bounding Well Flow Rate vs. Radius of Injected Bubble for Different Dip Angles and Constant Density Diff.

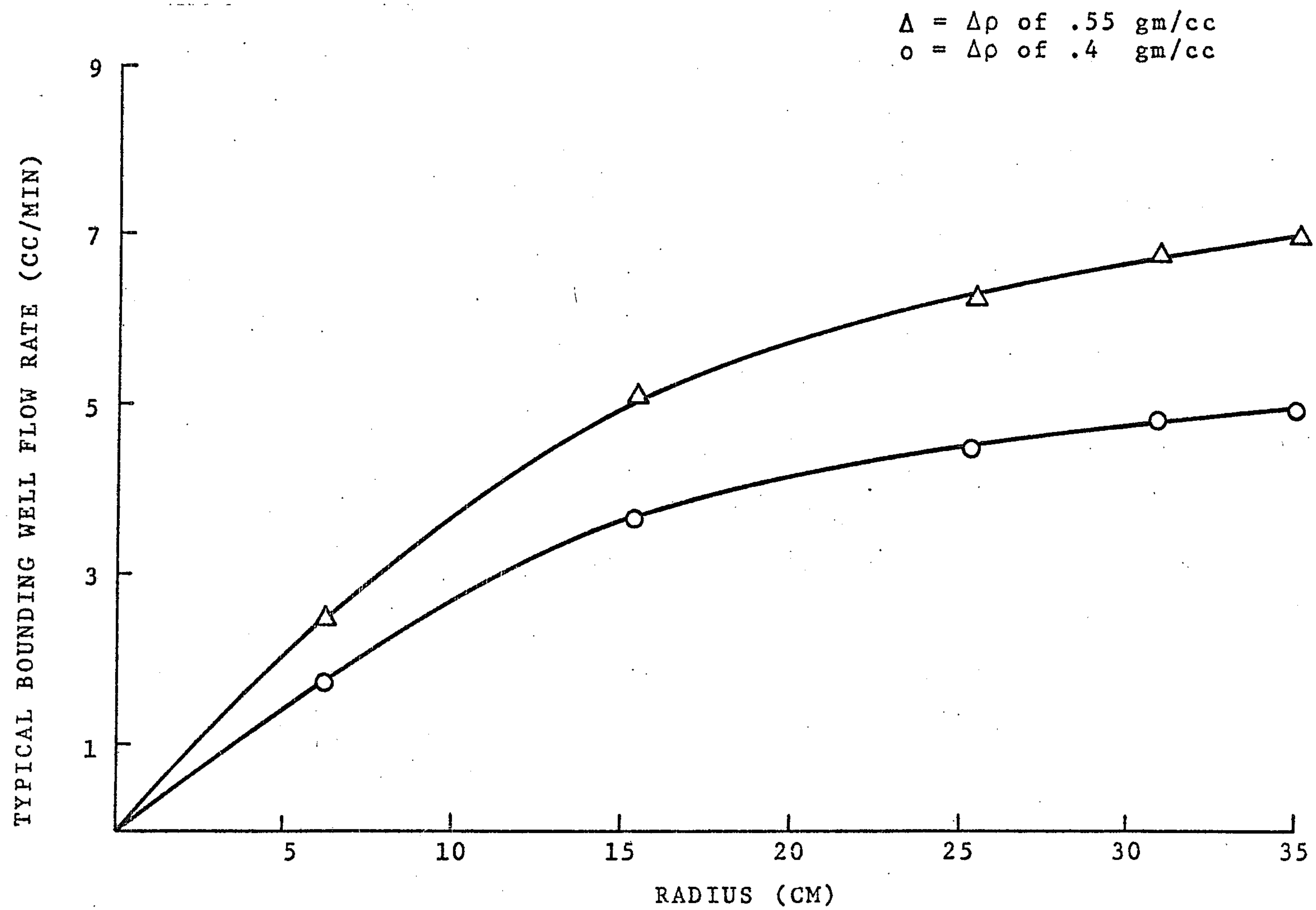


Figure 5.11 - Plot of Typical Bounding Well Flow Rate vs. Radius of Injected Bubble for Different Density Differences and Constant Dip

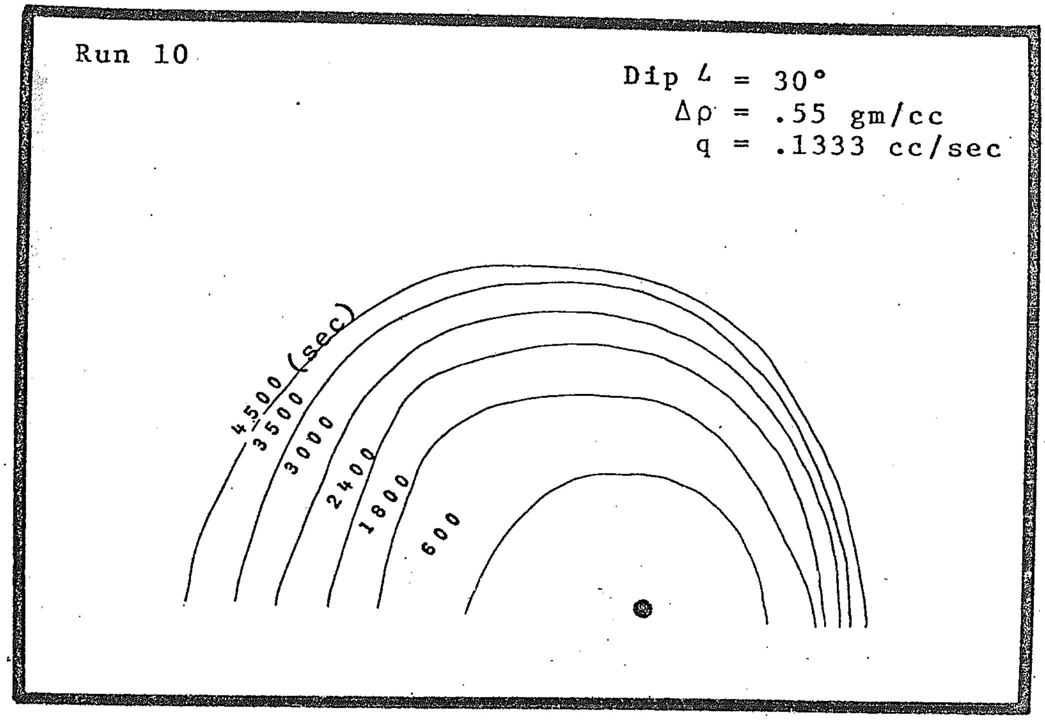


Figure 5.12 - Injection Phase of Run 10 (Bounding Wells Not Operating)

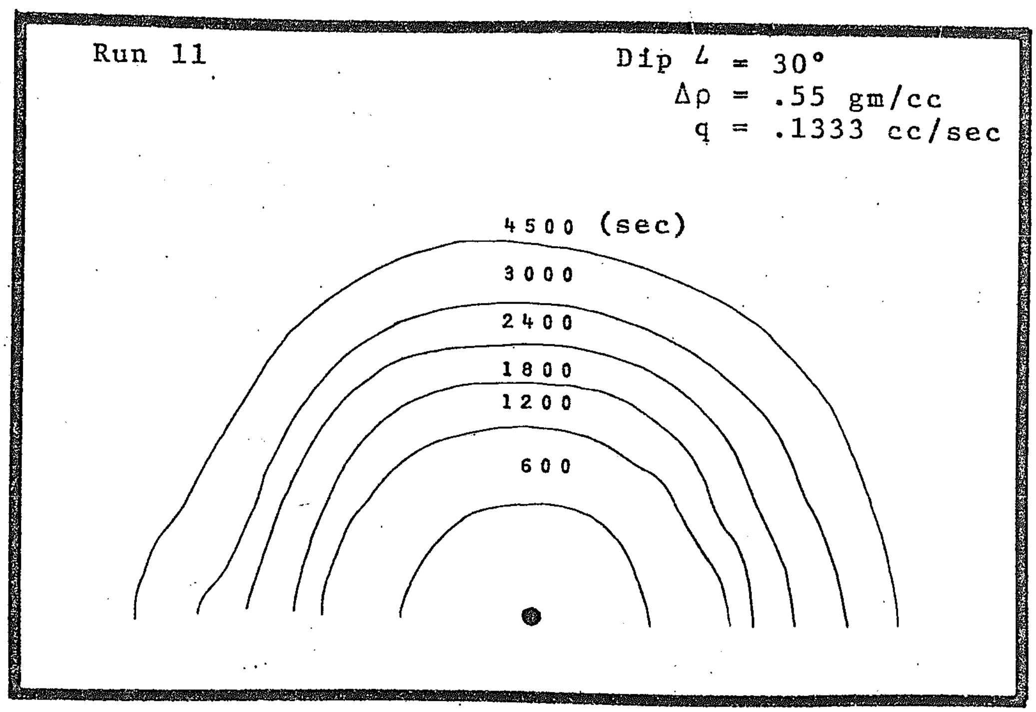


Figure 5.13 - Injection Phase of Run 11 (Bounding Wells Operating)

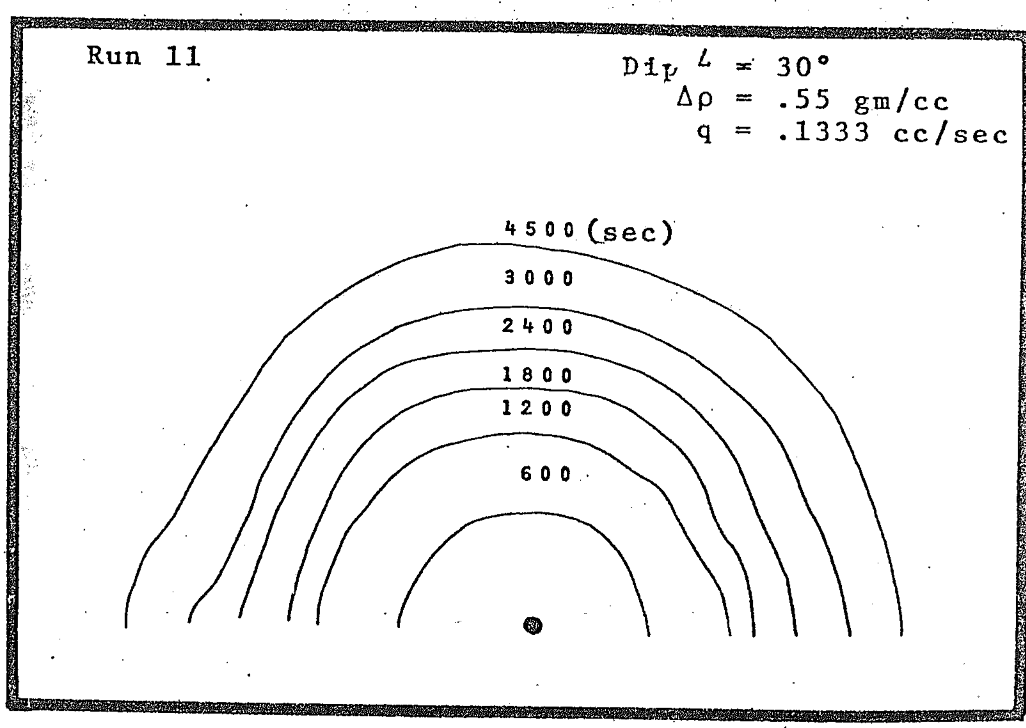


Figure 5.14 - Injection Phase of Run 11 (Bounding Wells Operating)

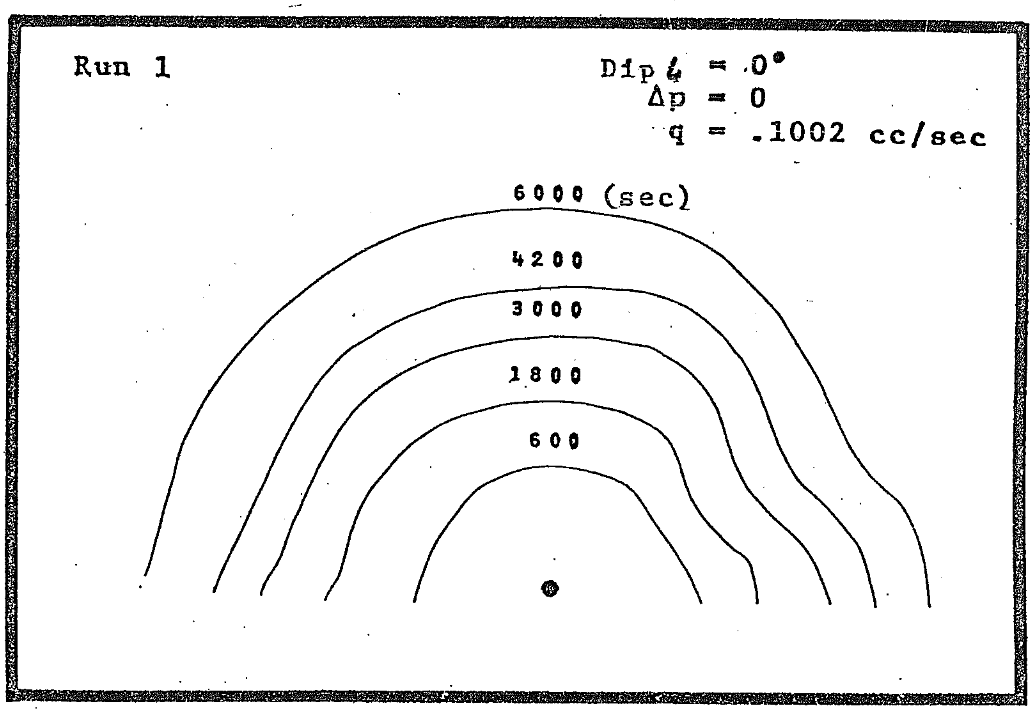


Figure 5.15 - Injection Phase of Run 1 (Bounding Wells Not Operating)

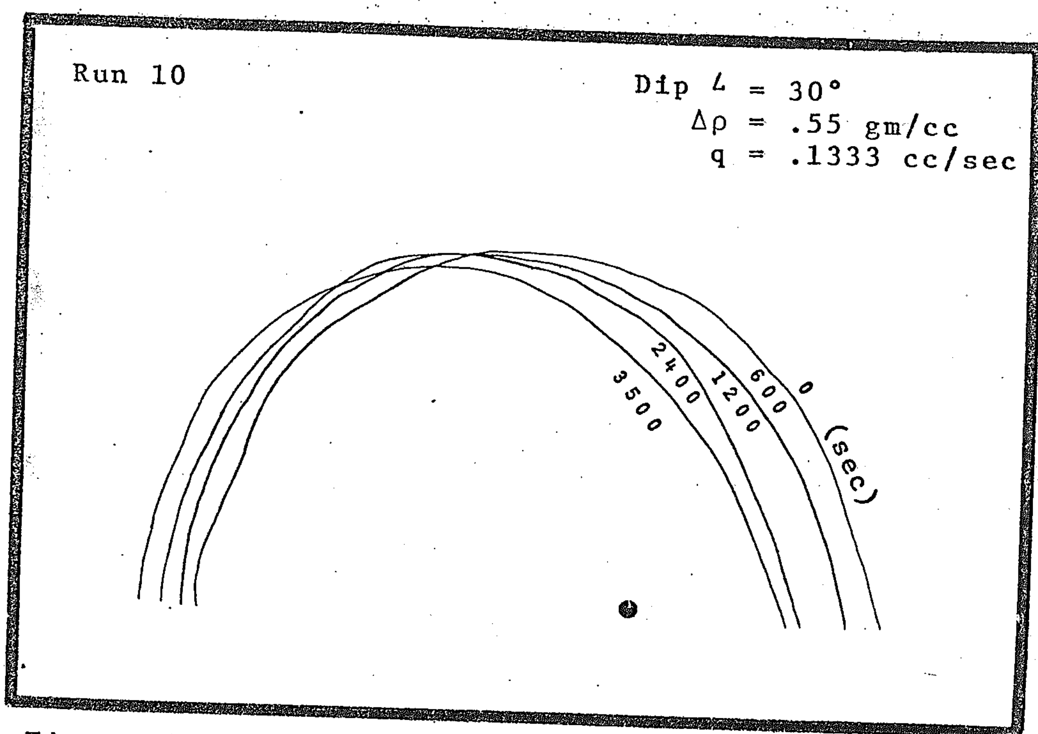


Figure 5.16 - Production Phase of Run 10 (Bounding Wells Not Operating)

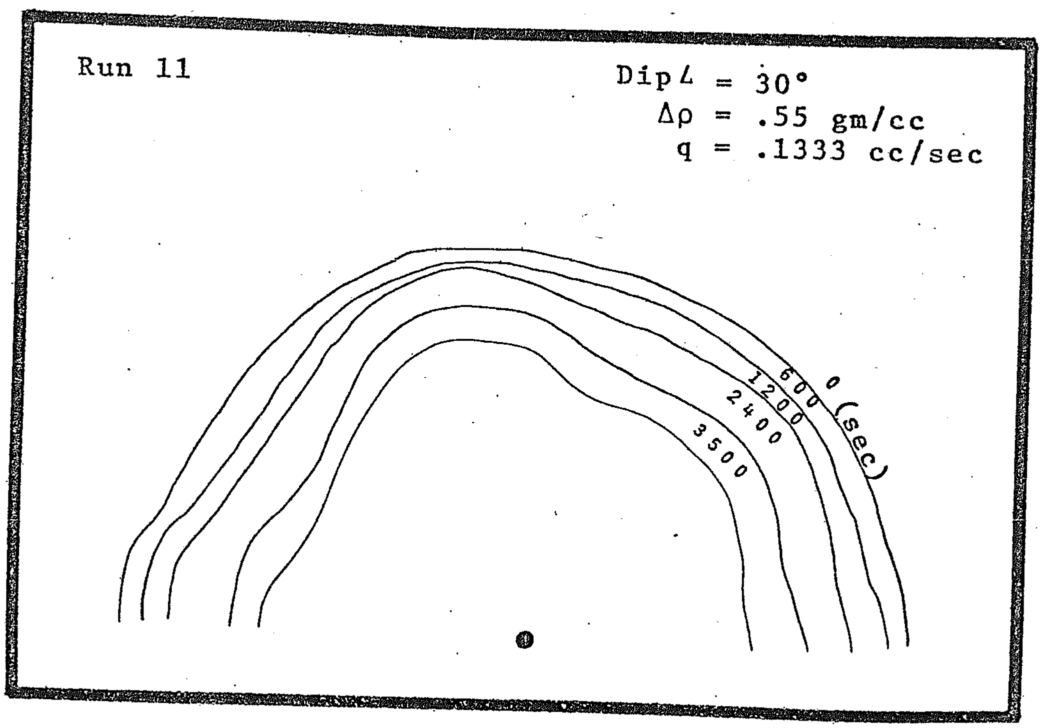


Figure 5.17 - Production Phase of Run Eleven (Bounding Wells Operating)

ten and eleven. Breakthrough occurs in run ten long before it does in run eleven. Run eleven behaves much like the horizontal system with no pre-existing fluid movement indicating that effects due to gravity have been negated. Figure 5.18 and 5.19 are a comparison between the production phases of runs one and eleven.

5.4 Recovery Efficiency

Runs and the accompanying recovery efficiencies are listed in Table 5.2.

The data in Table 5.2 show that the largest recovery efficiency to be expected is eighty-six percent for a horizontal aquifer with no flux, negliable density difference and no static storage period. By comparison a system dipping thirty degrees with a density difference between the native fluid and the injected fluid of .55 gm/cc the recovery efficiencies was thirty-three percent when no bounding wells were operated in the same system. The recovery efficiency increased to seventy-eight percent when bounding wells were operated in the same system. This compares favorably with the eighty-six percent recovery efficiency for the base case described above. Runs which included a static storage period yielded poorer recovery efficiencies. This result while partly due to diffusion, is primarily due to gravitational segregation. The mathematical model treated the system as if it were two-dimensional. At the higher density difference the actual three dimensional nature of the mini-aquifer becomes increasingly apparent because of a gravitational laydown of the displacement front. Although

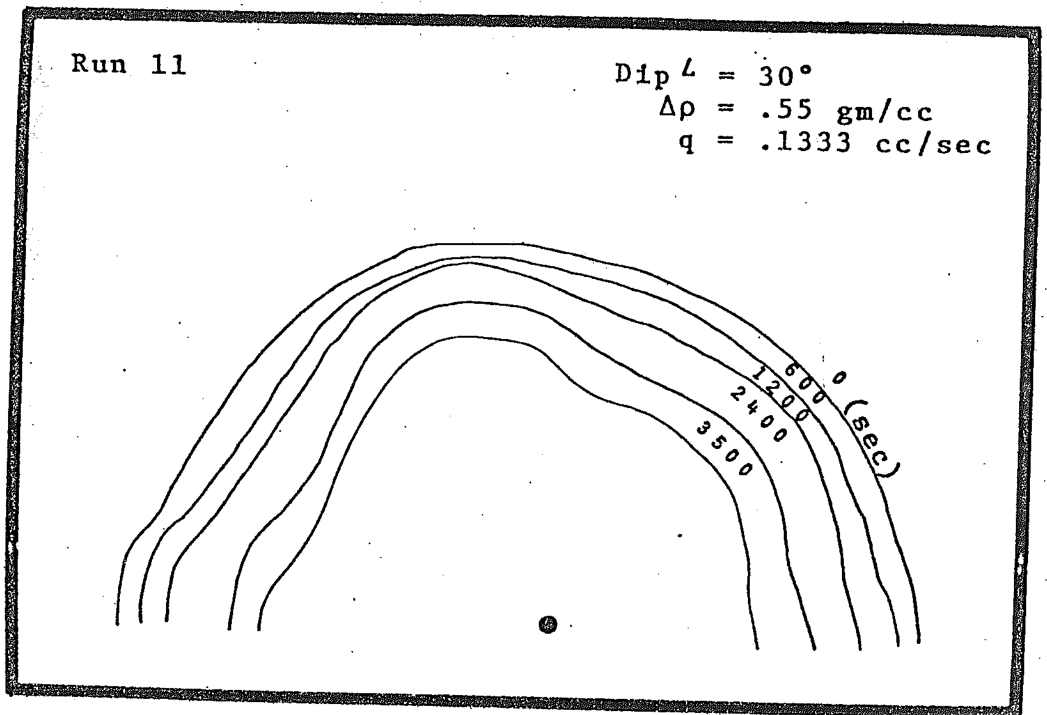


Figure 5.18 - Production Phase of Run Eleven (Bounding Wells Operating)

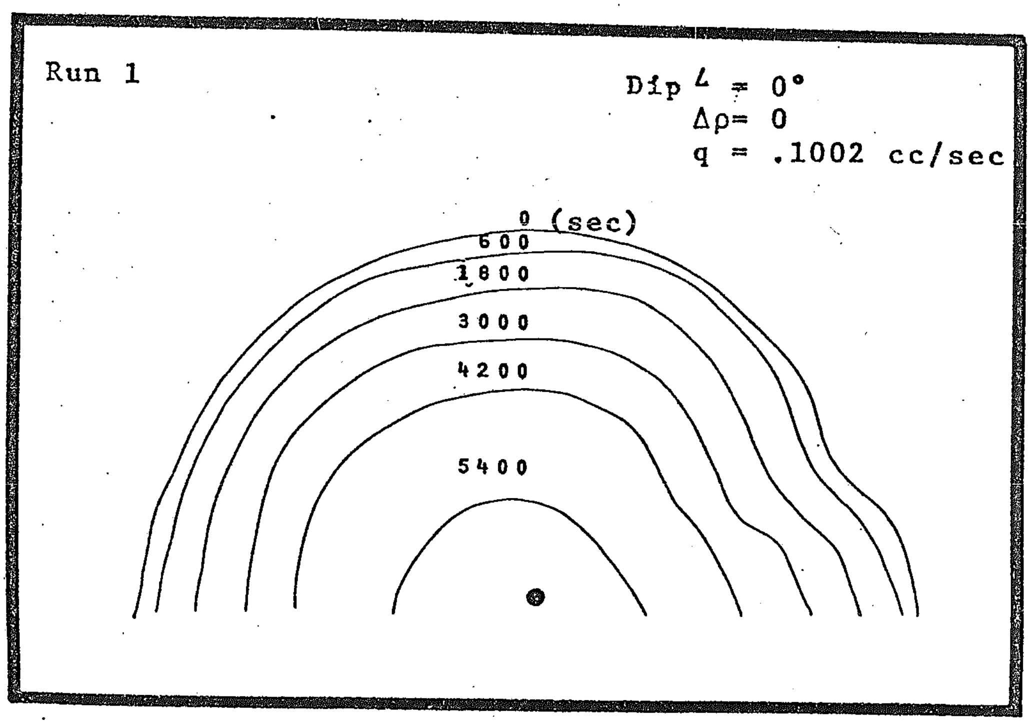


Figure 5.19 - Production Phase of Run One (Bounding Wells Not Operating)

Run No	Dip Angle Degrees	$\Delta\rho$	Stored Time (sec)	Recovery Efficiency %	Bounding Wells Operating?
10	30	.55	0	33	No
11	30	.55	0	78	Yes
12	30	.55	5400	41.6	Yes
13	30	.55	5400	38.9	Yes
14	0	.55	5400	54.25	No
15*	0	.02	0	86	No

*From Esmail[6]

Table 5.2 - Recovery Efficiencies

there was no bubble migration during the storage time (see Figure 5.16), the recovery efficiency was reduced to forty percent. In comparison (see Figure 5.18), the horizontal run for the same fluid yielded a recovery efficiency of fifty-four percent. Therefore, by the use of bounding wells, the dipping system was made seventy-five percent as efficient as the horizontal system, under the same conditions. If no bounding wells had been used, the recovery efficiency would have been zero.

5.5 Comments on Results

This study investigated the use of bounding wells to offset the effects of gravity due to density differences between injected and native fluids in dipping aquifers. Although the study was initiated with fresh water storage as its main consideration, the results may be applied to other miscible displacements in a dipping aquifer assuming the two fluids involved have different densities but approximately the same viscosities.

The use of bounding wells in water storage prospects in which either pre-existing groundwater movement or dip is present will always lead to improved recovery efficiencies. Where pre-existing groundwater movement is the problem, the computation of bounding well rates is straightforward and the procedure can probably be applied in field practice. The operation of bounding wells to negate the adverse effects of dip appears to show much promise, but the

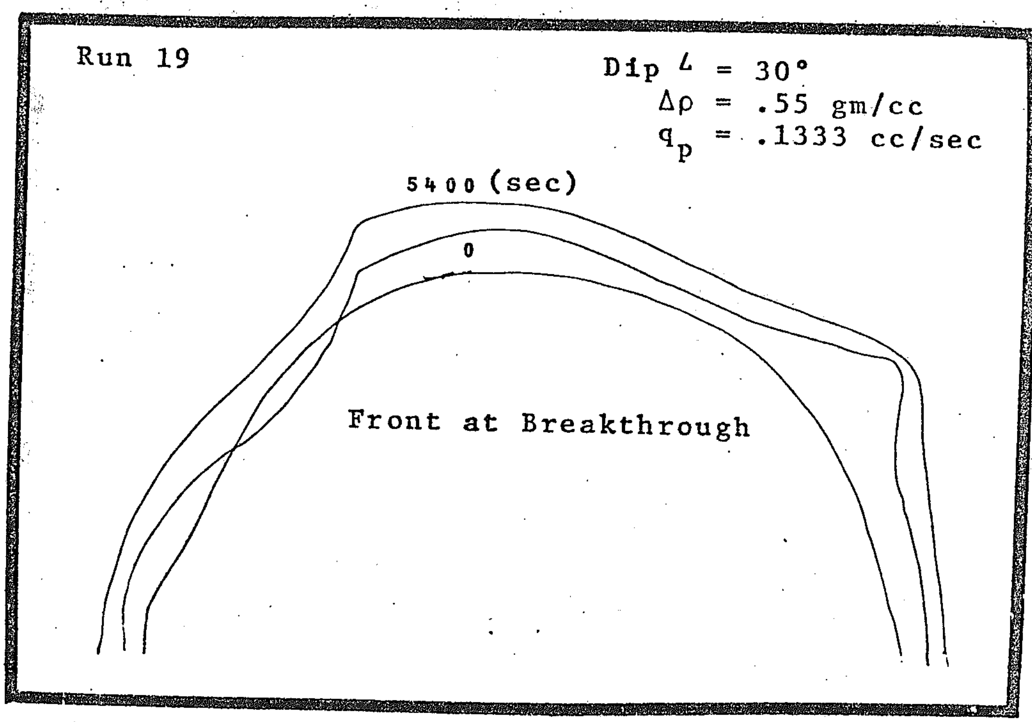


Figure 5.20 - Stored Bubble Dipping Aquifer (Bounding Wells Operating)

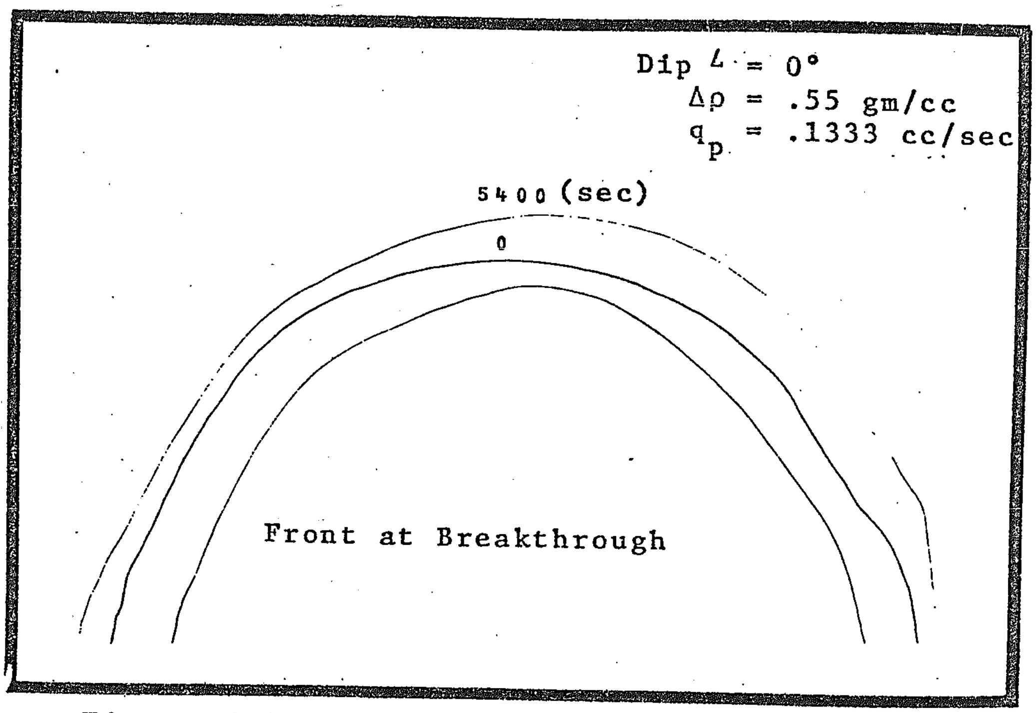


Figure 5.21 - Stored Bubble Horizontal Aquifer (Bounding Wells Not Operating)

computational procedure presented should be tested in thicker physical systems prior to application in the field.

CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

As a result of this investigation, the following conclusions can be drawn with respect to the storage of fresh water in saline aquifers.

(1) The bounding well flow rates predicted by a modification of the computational procedure developed by Whitehead and Langhetee[12] can successfully negate the deleterious effects of flux in a bounded physical system.

(2) For a given flux gradient bounding well rates are independent of bubble size and remain constant throughout the injection production storage cycle.

(3) The effects of gravity in a dipping aquifer can be substantially reduced by the use of a system of bounding wells.

(4) The bounding well rates in a dipping system are directly proportional to dip angle, density difference and radius of injected bubble. Hence bounding well flow rates for a dipping aquifer system constantly change with bubble radius during both injection and production.

(5) The bounding well flow rates computed using the procedures presented result in a balanced system hence the environmental impact of a bounding well system will be negligible.

(6) The results of this study are applicable to miscible processes other than fresh water storage in saline aquifers.

6.2 Recommendations

(1) The conclusion reached in this study should be verified in other physical systems.

(2) The effect of aquifer thickness in dipping systems should be investigated.

(3) The combined effect of flux and dip acting concurrently should be studied.

NOMENCLATURE

English

A = cross sectional area (cm^2) in equation 1.

a = the matrix of coefficients of the unknown flow rates in equation 8.

T = the transpose of matrix A

B = the matrix of values on the right side of equation 8

C = level setting constant

g = acceleration due to gravity (cm/sec^2)

h = aquifer thickness, cm

K = permeability, darcies

L = length, cm

M = pre-existing potential gradients, atm/cm

n = exponent

N = the number of bounding wells

P = desired isopotential value, atm

Q = solution matrix of unknown bounding well flow rates

q = volumetric flow rate (cc/sec)

r = radius, cm

r_e = length to nearest isopotential, cm

t = time, scc

V_g = velocity due to gravity, cm/sec

Greek

π = constant 3.14159

Φ = potential, atm

ϕ = porosity, fraction

μ = viscosity, centipoise

α = dip angle, radians

ρ = density gm/cc

$\Delta\rho$ = density difference gm/cc

Subscripts

B = boundary

D = due to density

e = pre-existing

i = index in a summation procedure

x = "x" direction

y = "y" direction

SELECTED REFERENCES

1. Esmail, O.J., 1966: "Investigation of the Technical Feasibility of Storing Fresh Water in Saline Aquifers", M.S. Thesis, Department of Petroleum Engineering, Louisiana State University, Baton Rouge, La.

2. Esmail, O.J. and Kimbler, O.K., 1967: "Investigation of the Technical Feasibility of Storing Fresh Water in Saline Aquifers", Water Resources Research, Vol. 3, No.3, pp. 683-95.

3. Kumar, A., and Kimbler, O.K., 1970: "Effect of Dispersion Gravitational Segregation, and Formation Stratification on the Recovery of Fresh Water Stored in Saline Aquifers", Water Resources Research, Vol.6, pp. 1689-1700.

4. Francis, J.L., Jr., 1970: "An Investigation of the Factors Influencing the Storage of Fresh Water in Inclined Saline Aquifers", M.S. Thesis, Department of Petroleum Engineering, Louisiana State University, Baton Rouge, La.

5. Painter, T.R., 1971: "Unequal Density Miscible Displacements in Thin Homogeneous Tilted Beds", M.S. Thesis, Department of Petroleum Engineering, Louisiana State University, Baton Rouge, La.

6. Esmail, W.J., 1973: "The Effect of Flux and Gravitational Forces on the Miscible Displacement in a Thin Homogeneous Bed", M.S. Thesis, Department of Petroleum Engineering, Louisiana State University, Baton Rouge, La.

7. Kimbler, O.K., Kazmann, R., and Whitehead, W.R., 1973: "Saline Aquifers - Future Storage Reservoirs for Fresh Water?", Vol.1 of preprints, Second International Symposium on Underground Waste Management and Artificial Recharge, New Orleans, La.

8. D'Amico, J.A., 1975: "Effect of Dip on the Subsurface Storage of Disposal of Fluids in Saline Aquifers" M.S. Thesis, Department of Petroleum Engineering, Louisiana State University, Baton Rouge, La.

9. Langhetee, E.J., 1974: "The Use of Bounding Wells to Control Flux in Underground Water Storage Projects", M.S. Thesis, Department of Petroleum Engineering, Louisiana State University, Baton Rouge, La.

10. Tate, P.T., 1976: "Effect of Dip on the Storage of Fresh Water (Or the Disposal of Waste) in a Saline Aquifer", M.S. Thesis, Department of Civil Engineering, Louisiana State University, Baton Rouge, La.

11. Agrawal, B.K., 1975: "Effect of Viscosity Ratio on the Recovery of Fresh Water Stored in Saline Aquifers", M.S. Thesis, Department of Petroleum Engineering, Louisiana State University, Baton Rouge, La.

12. Whitehead, W.R., and Langhetee, E.J., 1975: "Use of Bounding Wells to Counteract the Effects of Preexisting Groundwater Movement", Technical Paper, Louisiana State University, Baton Rouge, La.

13. Cederstrom, D.J., 1957: Geology and Groundwater Resources of the York James Peninsula, Virginia, The Commonwealth, December, 1947.

14. Whitehead, W.R., 1974: "Storage of Fresh Water in Saline Aquifers Using a Well Field", PhD Dissertation, Department of Civil Engineering, Louisiana State University, Baton Rouge, La.

15. Harpaz, Y., and Bear, J., 1964: "Investigation on Mixing of Waters in Underground Storage Operations" Int. Assn. Sci. Hydrol., Pub.G4, pp.132-153.

16. Collins, R.E., 1961: "Flow of Fluids Through Porous Materials", Reinhold Publishing Corp, New York.

17. Private Communication, Dr. Walter R. Whitehead, Petroleum Engineering Department, Louisiana State University, Baton Rouge, La.

18. Craft, B.C. and Hawkins, M.F., 1959: "Applied Petroleum Reservoir Engineering", Prentice-Hall, Inc., Englewood Cliffs, N.J.

19. Lin, J.K., 1972: "An Image Well Method for Bounding Arbitrary Reservoir Shapes in the Streamline Model", PhD Dissertation, University of Texas at Austin.

APPENDIX A

ASSUMPTIONS - LIMITATIONS OF COMPUTER PROGRAM

There are certain assumptions inherent in the computer program and they are as follows:

(1) Porous and permeable reservoir

(2) Horizontal

(3) Isotropic

(4) Flux and dip are in the negative y direction

(5) Incompressible fluids

(6) Mobility ratio of one

Limitations:

(1) Only for circular storage system

(2) Injection well is at the center of the storage circle

(3) The program is written for a finite square system, bounded by two no flow boundaries, and two iso-potentials.

APPENDIX B

COMPUTER LISTINGS

Although the program is believed to be correct consistent with the assumptions and the approximations involved, neither the author, Louisiana State University, nor the Louisiana Water Resources Research Institute make any claims concerning its validity or correctness and anyone making use of the program does so at his own risk.

```
C
C     THOMAS E. WILLIAMS
C     LOUISIANA STATE UNIVERSITY, BATON ROUGE
C     WATER RESOURCES -1978-
C
C
C     THE USE OF BOUNDING WELLS TO NEGATE FLUX IN HORIZONTAL AQUIFERS
C
C
C
C    PROGRAM TO CALCULATE BOUNDING-WELL RATES NECESSARY TO NEGATE PRE-
C    EXISTING GROUNDWATER FLOW BY CREATING A POTENTIOMETRIC
C     PLATEAU. COORDINATE AXES MUST BE SET UP SO THAT ORIGIN IS AT THE
C     CENTER OF THE POTENTIOMETRIC PLATEAU AND THE PRE-EXISTING
C     GROUNDWATER MOVEMENT IS IN THE NEGATIVE Y DIRECTION.
C
C
C
C    ******************************************************************
C
C
C     DATA TO BE READ IN FOR HORIZONTAL AQUIFER EXPERIENCING FLUX
C
C
C   FIRST CARD  = FORMAT(3F12.0,2I3)
C     RBOUND    = RADIUS OF POTENTIOMETRIC PLATEAU. (FEET)
C     ADFT      = DISTANCE TO NO FLOW BOUNDARY, FEET
C     BDFT      = DISTANCE TO ISOPOTENTIAL, FEET
C     NBW       = NUMBER OF BOUNDING WELLS
C     NRC       = NUMBER OF ROWS AND COLUMNS TO BE USED FOR IMAGING
C
C   SECOND CARD = FORMAT(6F12.0)  (THREE WELL LOCATIONS PER CARD)
C      XBWFT(I) = X COORDINATE OF ITH BOUNDING WELL. (FEET)
C      YBWFT(I) = Y COORDINATE OF ITH BOUNDING WELL. (FEET)
C
C   THIRD CARD  = FORMAT(4F12.0)
C      PLYMEI   = AQUIFER PERMEABILITY. (MEINZERS)
C      HFT      = AQUIFER THICKNESS. (FEET)
C      SG       = SPECIFIC GRAVITY OF NATIVE FLUID. (20C/20C)
C      PGRAD    = PRE-EXISTING POTENTIOMETRIC GRADIENT. (FEET/MILE)
C
C    ******************************************************************
C
C
C    DEFINITION OF VARIABLE NAMES USED IN PROGRAM.
C     XSTART    = STARTING POINT FOR FINDING 'X' COORDINATES OF IMAGE WELLS
C
C      A(I,J)   = ELEMENTS OF THE A MATRIX IN EQUATION 11.
C      ATA(I,J) = ELEMENTS OF THE MATRIX OBTAINED BY PRE-MULTIPLYING
C                   THE A MATRIX BY ITS TRANSPOSE.
C      ATB(I)   = ELEMENTS OF THE MATRIX OBTAINED BY PRE-MULTIPLYING
C                   THE B MATRIX BY THE TRANSPOSE OF THE A MATRIX.
C      AD       = DISTANCE TO NO FLOW BOUNDARY, CM
C      B(I)     = ELEMENTS OF THE B MATRIX IN EQUATION 11.
C      BD       = DISTANCE TO ISOPOTENTIAL, CM
C      CFFTCM   = CONVERSION FACTOR. (CM/FT)
C      CFFMAC   = CONVERSION FACTOR. ((ATMOSPHERES/CM)/(FEET/MILE))
C      CONST1   = A CONSTANT USED IN COMPUTING THE ELEMENTS OF THE
C                  B MATRIX.
```

```
C        DTHETA    = INCREMENT BY WHICH THE ANGLE THETA IS INCREASED
C                    DURING COMPUTATION OF BOUNDARY POINT
C                    COORDINATES. (RADIANS)
C        H         = AQUIFER THICKNESS. (CM)
C        ICHECK    = A FIXED POINT NUMBER USED AS A COUNTER.
C        NBP       = NUMBER OF BOUNDARY POINTS.
C        NNRC      = COUNTER IN 'A' MATRIX
C        PGRADA    = PRE-EXISTING POTENTIOMETRIC GRADIENT. (ATMS/CM)
C        QBW(I)    = RATE OF THE ITH BOUNDING WELL. (CC/SEC)
C        QBWGPM    = RATE OF A BOUNDING WELL. (GPM)
C        R         = RADIUS OF POTENTIOMETRIC PLATEAU. (CM)
C        SOLVE     = A SUBROUTINE FOR SOLVING A N X N SET OF LINEAR
C                    EQUATIONS.
C        THETA     = ANGLE USED TO COMPUTE COORDINATES OF BOUNDARY POINTS.
C                    ANGLE IS MEASURED CLOCKWISE FROM THE POSITIVE Y
C                    AXIS. (RADIANS)
C        XBP(I)    = X COORDINATE OF ITH BOUNDARY POINT. (CM)
C        XBW(I)    = X COORDINATE OF ITH BOUNDING WELL. (CM)
C        XNBP      = FLOATING POINT VALUE OF NBP.
C        XNUM      = THE SQUARE OF THE DISTANCE BETWEEN A BOUNDARY POINT
C        XNRC      = COUNTER IN 'A' MATRIX
C        XSTART    = STARTING POINT FOR FINDING 'X' COORDINATES OF IMAGE
C                    WELLS
C        XW(L)     = ARRAY CONTAINING 'X' COORDINATES OF IMAGE WELLS
C                    AND A BOUNDING WELL. (SQ CM)
C        YBP(I)    = Y COORDINATE OF ITH BOUNDARY POINT. (CM)
C        YBW(I)    = Y COORDINATE OF ITH BOUNDING WELL. (CM)
C        YSTART    = STARTING POINT FOR FINDING 'Y' COORDINATES OF IMAGE
C                    WELLS
C        YW(K)     = ARRAY CONTAINING 'Y' COORDINATES OF IMAGE WELLS
C
C
C    ******************************************************************
C
C
C
C
C      COMPUTER CARDS FOR HORIZONTAL AQUIFER EXPERIENCING FLUX
C
C
      IMPLICIT REAL*8(A-H),REAL*8(O-Z)
      DIMENSION XBWFT(20),YBWFT(20),XBW(20),YBW(20),XBP(60),
     1          YBP(60),A(60,20),B(60),P(60),XW(300),YW(300)
      COMMON ATA(20,20),ATB(20),QBW(20),ICHECK
C
C
C      READING IN DATA (FOR HORIZONTAL AQUIFER)
C      ----------------------------------------
C
   20 READ(5,10000,END=65) RBOUND, ADFT,BDFT,NBW,NRC
      READ(5,11000) (XBWFT(I),YBWFT(I),I=1,NBW)
      READ(5,12000) PLYMEI,HFT,SG,PGRAD
C
      NBP=3*NBW
C
C
C     PRINTING DATA
C     -------------
C
      WRITE(6,13000) RBOUND,NBW,PLYMEI,HFT,SG,PGRAD
```

```
C
C
C     CONVERSION FACTORS (FIELD UNITS TO C.G.S. UNITS)
C     ------------------------------------------------
C
      CFFTCM=30.4801
      H=HFT*CFFTCM
      R=RBOUND*CFFTCM
      AD=ADFT*CFFTCM
      BD=BDFT*CFFTCM
      DO25I=1,NBW
      XBW(I)=XBWFT(I)*CFFTCM
      YBW(I)=YBWFT(I)*CFFTCM
   25 CONTINUE
C
C
C     GENERATING COORDINATES OF BOUNDARY POINTS
C     -----------------------------------------
C
      XNBP=NBP
      DTHETA=6.283185/XNBP
      THETA=0.0
      DO30I=1,NBP
      YBP(I)=R*DCOS(THETA)
      XBP(I)=R*DSIN(THETA)
      THETA=THETA+DTHETA
   30 CONTINUE
C
C
C     GENERATING THE 'A' MATRIX
C     -------------------------
C
      NNRC=NRC-1
      XNRC=NRC+3
      XSTART=-AD*XNRC
      YSTART=-BD*XNRC
      DO35I=1,NBP
      DO35J=1,NBW
      SUM1=0.0
      SUM2=0.0
      DO32K=1,NRC,2
      YM1=2*(K+1)
      YW(K)=YSTART+YM1*BD+YBW(J)
      DO31L=1,NRC,2
      XM1=2*(L+1)
      XW(L)=XSTART+XM1*AD+XBW(J)
      XNUM=(XBP(I)-XW(L))**2+(YBP(I)-YW(K))**2
      SUM1=SUM1+DLOG(XNUM)
   31 CONTINUE
      DO32M=2,NNRC,2
      XM2=2*(M+1)
      XW(M)=XSTART+XM2*AD-XBW(J)
      XNUM=(XBP(I)-XW(M))**2+(YBP(I)-YW(K))**2
      SUM1=SUM1+DLOG(XNUM)
   32 CONTINUE
      DO34KK=2,NNRC,2
      YM2=2*(KK+1)
      YW(KK)=YSTART+YM2*BD-YBW(J)
      DO33LL=1,NRC,2
      XM3=2*(LL+1)
```

```
      XW(LL)=XSTART+XM3*AD+XBW(J)
      XNUM=(XBP(I)-XW(LL))**2+(YBP(I)-YW(KK))**2
      SUM2=SUM2+DLOG(XNUM)
   33 CONTINUE
      DO34MM=2,NNRC,2
      XM4=2*(MM+1)
      XW(MM)=XSTART+XM4*AD-XBW(J)
      XNUM=(XBP(I)-XW(MM))**2+(YBP(I)-YW(KK))**2
      SUM2=SUM2+DLOG(XNUM)
   34 CONTINUE
      A(I,J)=SUM1-SUM2
   35 CONTINUE
C
C
C     GENERATING THE 'B' MATRIX
C     -------------------------
C
      CONST=(PLYMEI*H*PGRAD)/8910083.524
      DO40 I=1,NBP
      B(I)=CONST*YBP(I)
 40   CONTINUE
C
C
C     GENERATING 'A TRANSPOSE' * 'A'
C     ------------------------------
C
      DO50I=1,NBW
      DO50J=1,I
      ATA(J,I)=0.0
      DO45K=1,NBP
      ATA(J,I)=ATA(J,I)+A(K,I)*A(K,J)
   45 CONTINUE
      ATA(I,J)=ATA(J,I)
   50 CONTINUE
C
C
C     GENERATING 'A TRANSPOSE' * 'B'
C     ------------------------------
C
      DO55I=1,NBW
      ATB(I)=0.0
      DO55J=1,NBP
      ATB(I)=ATB(I)+B(J)*A(J,I)
   55 CONTINUE
C
C
C     SOLVING FOR THE BOUNDING WELL FLOW RATES
C     ----------------------------------------
C
      CALL SOLVE (NBW)
      IF(ICHECK.EQ.1) GOTO20
C
C
C     PRINTING THE RESULTS
C     --------------------
C
      WRITE(6,14000)
      DO60I=1,NBW
      QBWCCM=QBW(I)*60.00
      WRITE(6,1500) I,XBWPT(I),YBWPT(I),QBWCCM
```

```
   60 CONTINUE
      WRITE(6,20000)NRC
      GO TO 20
C
C
C     FORMAT STATEMENTS
C     -----------------
C
10000 FORMAT(3F12.0,2I3)
11000 FORMAT(6F12.0)
12000 FORMAT(4F12.0)
13000 FORMAT(1H1,29X,'DATA'/,30X,'----'///,
     1  6X,'RADIUS OF POTENTIOMETRIC PLATEAU (FEET)',6X,F9.4/,
     2  6X,'NUMBER OF BOUNDING WELLS',22X,I3/,
     3  6X,'AQUIFER PERMEABILITY (MEINZERS)',14X,F9.4/,
     4  6X,'AQUIFER THICKNESS (FEET)',21X,F9.4/,
     5  6X,'SPECIFIC GRAVITY OF NATIVE FLUID. (20C/20C)',2X,F9.4/,
     6  6X,'PRE-EXISTING FLOW GRADIENT (FEET/MILE)',7X,F9.4////)
14000 FORMAT(25X,'BOUNDING WELL DATA',/,25X,'-------------------',///,
     1  6X,'WELL NO.',5X,'X(FEET)',9X,'Y(FEET)',9X,'RATE(CCPM)'//)
 1500 FORMAT(8X,I3,7X,F8.5,8X,F8.5,10X,E15.8)
20000 FORMAT(///,'FOR NRC EQUAL',I3)
C
   65 STOP
      END
```

```
C       THOMAS E. WILLIAMS
C       LOUISIANA STATE UNIVERSITY,BATON ROUGE
C       WATER RESOURCES -1978-
C
C
C       THE USE OF BOUNDING WELLS TO NEGATE THE EFFECTS OF GRAVITY IN
C       DIPPING AQUIFERS
C
C
C      PROGRAM TO CALCULATE BOUNDING-WELL RATES NECESSARY TO NEGATE
C       GRAVITY EFFECTS BY CREATING A POTENTIOMETRIC PLATEAU.COORDINATE
C       AXES MUST BE SET UP SO THAT THE ORIGIN IS AT THE CENTER OF THE
C       POTENTIOMETRIC PLATEAU AND DIP IS IN THE NEGATIVE Y DIRECTION
C
C
C
C       ******************************************************************
C
C
C       DATA FOR DIPPING AQUIFER
C
C
C     FIRST CARD  = FORMAT(3F12.0,2I3)
C       RBOUND    = RADIUS OF POTENTIOMETRIC PLATEAU
C       ADFT      = DISTANCE TO NO FLOW BOUNDARY, FEET
C       BDFT      = DISTANCE TO ISOPOTENTIAL, FEET
C       NBW       = NUMBER OF BOUNDING WELLS.
C       NRC       = NUMBER OF ROWS AND COLUMNS TO BE USED FOR IMAGING
C
C     SECOND CARD = FORMAT(6F12.0) (THREE WELL LOCATIONS PER CARD)
C       XBWFT     = X COORDINATE OF ITH BOUNDING WELL. (FEET)
C       YBWFT     = Y COORDINATE OF ITH BOUNDING WELL (FEET)
C
C     THIRD CARD  = FORMAT(4F12.0)
C         PLYMEI  = AQUIFER PERMEABILITY. (MEINZERS)
C         HFT     = AQUIFER THICKNESS. (FEET)
C         SG      = SPECIFIC GRAVITY OF NATIVE FLUID. (20C/20C)
C
C     LAST CARD   = FORMAT(4F12.0)
C       DANG      = ANGLE OF DIP IN AQUIFER
C       RE        = DISTANCE TO NEAREST ISOPOTENTIAL
C       DD        = DENSITY DIFFERENCE
C       ZN        = EXPONENT TERM
C
C
C      ******************************************************************
C
C
C      DEFINITION OF VARIABLE NAMES USED IN PROGRAM.
C
C
C         A(I,J)    = ELEMENTS OF THE A MATRIX IN EQUATION 11.
C         ATA(I,J)  = ELEMENTS OF THE MATRIX OBTAINED BY PRE-MULTIPLYING
C                       THE A MATRIX BY ITS TRANSPOSE.
C         AD        = DISTANCE TO NO FLOW BOUNDARY, CM.
C         ATB(I)    = ELEMENTS OF THE MATRIX OBTAINED BY PRE-MULTIPLYING
C                       THE B MATRIX BY THE TRANSPOSE OF THE A MATRIX.
C         BD        = DISTANCE TO ISOPOTENTIAL, CM
C         B(I)      = ELEMENTS OF THE B MATRIX IN EQUATION 11.
```

```
C        CFFTCM    = CONVERSION FACTOR. (CM/FT)
C        CFPMAC    = CONVERSION FACTOR. ((ATMOSPHERES/CM)/(FEET/MILE))
C        CONST1    = A CONSTANT USED IN COMPUTING THE ELEMENTS OF THE
C                     B MATRIX.
C        DP        = CHANGE IN POTENTIAL AT BOUNDARY DUE TO GRAVITY EFFCTS
C        DTHETA    = INCREMENT BY WHICH THE ANGLE THETA IS INCREASED
C                     DURING COMPUTATION OF BOUNDARY POINT
C                     COORDINATES. (RADIANS)
C        H         = AQUIFER THICKNESS. (CM)
C        ICHECK    = A FIXED POINT NUMBER USED AS A COUNTER.
C        NBP       = NUMBER OF BOUNDARY POINTS.
C        NNRC      = COUNTER IN 'A' MATRIX
C        QBW(I)    = RATE OF THE ITH BOUNDING WELL. (CC/SEC)
C        QBWGPM    = RATE OF A BOUNDING WELL. (GPM)
C        R         = RADIUS OF POTENTIOMETRIC PLATEAU. (CM)
C        SOLVE     = A SUBROUTINE FOR SOLVING A N X N SET OF LINEAR
C                     EQUATIONS.
C        THETA     = ANGLE USED TO COMPUTE COORDINATES OF BOUNDARY POINTS.
C                     ANGLE IS MEASURED CLOCKWISE FROM THE POSITIVE Y
C                     AXIS. (RADIANS)
C        VG        = VELOCITY DUE TO GRAVITY
C        XBP(I)    = X COORDINATE OF ITH BOUNDARY POINT. (CM)
C        XBW(I)    = X COORDINATE OF ITH BOUNDING WELL. (CM)
C        XNBP      = FLOATING POINT VALUE OF NBP.
C        XNUM      = THE SQUARE OF THE DISTANCE BETWEEN A BOUNDARY POINT
C                     AND A BOUNDING WELL. (SQ CM)
C        XNRC      = COUNTER IN 'A' MATRIX
C        XSTART    = STARTING POINT FOR FINDING 'X' COORDINATES OF IMAGE
C                    WELLS
C        XW(L)     = ARRAY CONTAINING 'X' COORDINATES OF IMAGE WELLS
C        YBP(I)    = Y COORDINATE OF ITH BOUNDARY POINT. (CM)
C        YBW(I)    = Y COORDINATE OF ITH BOUNDING WELL. (CM)
C        YSTART    = STARTING POINT FOR FINDING 'Y' COORDINATES OF IMAGE
C                    WELLS
C        YW(K)     = ARRAY CONTAINING 'Y' COORDINATES OF IMAGE WELLS
C
C
C    **********************************************************************
C
C
C      COMPUTER CARDS FOR DIPPING AQUIFER
C
C
      IMPLICIT REAL*8(A-H),REAL*8(O-Z)
      DIMENSION XBWFT(20),YBWFT(20),XBW(20),YBW(20),XBP(60),YBP(60),
     1                  A(60,20),B(60),P(60),XW(300),YW(300)
      COMMON ATA(20,20),ATB(20),QBW(20),ICHECK
C
C
C    READING IN DATA
C    ---------------
C
C
   20 READ(5,10000,END=65) RBOUND, ADFT,BDFT,NBW,NRC
      READ(5,11000) (XBWFT(I),YBWFT(I),I=1,NBW)
      READ(5,12000) PLYMEI,HFT,SG
      READ(5,20000)  DANG,RE,DD,ZH
C
C
      NBP=3*NBW
```

```
C
C
C     PRINTING DATA
C     -------------
C
C
      WRITE(6,13000)RBOUND,NBW,PLYMEI,HFT,SG
C
C     CONVERSION FACTORS (FIELD UNITS TO C.G.S. UNITS)
      CFFTCM=30.4801
      H=HFT*CFFTCM
      R=RBOUND*CFFTCM
      AD=ADFT*CFFTCM
      BD=BDFT*CFFTCM
      DO25I=1,NBW
      XBW(I)=XBWFT(I)*CFFTCM
      YBW(I)=YBWFT(I)*CFFTCM
   25 CONTINUE
C
C
C     GENERATING COORDINATES OF BOUNDARY POINTS
C     -----------------------------------------
C
C
      XNBP=NBP
      DTHETA=6.283185/XNBP
      THETA=0.0
      DO30I=1,NBP
      XBP(I)=R*DSIN(THETA)
      YBP(I)=R*DCOS(THETA)
      THETA=THETA+DTHETA
   30 CONTINUE
C
C
C     GENERATING THE 'A' MATRIX
C     -------------------------
C
C
      NNRC=NRC-1
      XNRC=NRC+3
      XSTART=-AD*XNRC
      YSTART=-BD*XNRC
      DO35I=1,NBP
      DO35J=1,NBW
      SUM1=0.0
      SUM2=0.
      DO32K=1,NRC,2
      YM1=2*(K+1)
      YW(K)=YSTART+YM1*BD+YBW(J)
      DO31L=1,NRC,2
      XM1=2*(L+1)
      XW(L)=XSTART+XM1*AD+XBW(J)
      XNUM=(XBP(I)-XW(L))**2+(YBP(I)-YW(K))**2
      SUM1=SUM1+DLOG(XNUM)
   31 CONTINUE
      DO32M=2,NNRC,2
      XM2=2*(M+1)
      XW(M)=XSTART+XM2*AD-XBW(J)
      XNUM=(XBP(I)-XW(M))**2+(YBP(I)-YW(K))**2
      SUM1=SUM1+DLOG(XNUM)
```

```
   32 CONTINUE
      DO34KK=2,NNRC,2
      YM2=2*(KK+1)
      YW(KK)=YSTART+YM2*BD-YBW(J)
      DO33LL=1,NRC,2
      XM3=2*(LL+1)
      XW(LL)=XSTART+XM3*AD+XBW(J)
      XNUM=(XBP(I)-XW(LL))**2+(YBP(I)-YW(KK))**2
      SUM2=SUM2+DLOG(XNUM)
   33 CONTINUE
      DO34MM=2,NNRC,2
      XM4=2*(MM+1)
      XW(MM)=XSTART+XM4*AD-XBW(J)
      XNUM=(XBP(I)-XW(MM))**2+(YBP(I)-YW(KK))**2
      SUM2=SUM2+DLOG(XNUM)
   34 CONTINUE
      A(I,J)=SUM1-SUM2
   35 CONTINUE
C
C
C      GENERATING THE 'B' MATRIX
C     ------------------------
C
C
      CONST1=.616521*(PLYMEI*H)
      VG=.000967*DD*DSIN(DANG)*(((R**2)/((RE**2))))**ZN
      DO 40 I=1,NBP
      DP=VG*YBP(I)
      B(I)=DP*CONST1
  40  CONTINUE
C
C
C GENERATING THE 'A TRANSPOSE' * 'A'
C     ------------------------------
C
C
      DO50 I=1,NBW
      DO50 J=1,I
      ATA(J,I)=0.0
      DO45 K=1,NBP
      ATA(J,I)=ATA(J,I)+A(K,I)*A(K,J)
  45  CONTINUE
      ATA(I,J)=ATA(J,I)
  50  CONTINUE
C
C
C GENERATING 'A TRANSPOSE' * 'B'
C     ----------------------------
C
C
      DO55I=1,NBW
      ATB(I)=0.0
      DO55J=1,NBP
      ATB(I)=ATB(I)+B(J)*A(J,I)
  55  CONTINUE
C
C
C SOLVING FOR THE BOUNDING WELL FLOW RATES
C     ------------------------------------
C
```

```
C
      CALL SOLVE (NBW)
C
C
C
C   PRINTING THE RESULTS
C       --------------------
C
C
      WRITE(6,1400)
      DO60I=1,NBW
      QBWCCM=QBW(I)*60.00
      WRITE(6,1500)I,XBWFT(I),YBWFT(I),QBWCCM
   60 CONTINUE
C
C
C   FORMAT STATEMENTS
C       --------------
C
C
10000 FORMAT(2F12.0,1I3)
11000 FORMAT(6F12.0)
12000 FORMAT(4F12.0)
13000 FORMAT(1H1,29X,'DATA'/,30X,'----'///,
     1  6X,'RADIUS OF POTENTIOMETRIC PLATEAU (FEET)',6X,F9.4/,
     2  6X,'NUMBER OF BOUNDING WELLS',22X,I3/,
     3  6X,'AQUIFER PERMEABILITY (MEINZERAS)',14X,F9.4/,
     4  6X,'AQUIFER THICKNESS (FEET)',21X,F9.4/,
     5  6X,'SPECIFIC GRAVITY OF NATIVE FLUID. (20C/20C)',2X,F9.4/,
     6  'PRE-EXISTING FLOW GRADIENT (FEET/MILE)', 7X,F9.4////)
1400  FORMAT(25X,'BOUNDING WELL DATA',/,25X,'-------------------',///,
     1  6X,'WELL NO.',5X,'X(FEET)',9X,'Y(FEET)',9X,'RATE(GPM)'//)
 1500 FORMAT(8X,I3,7X,F8.5,8X,F8.5,10X,E15.8)
20000 FORMAT(4F12.0)
   65 STOP
      END
```

```
      SUBROUTINE SOLVE (N)
C     ------------------------
C
C
      IMPLICIT REAL*8(A-H),REAL*8(O-Z)
      DIMENSION A(20,21),ID(20)
      COMMON ATA(20,20),ATB(20),QBW(20),ICHECK
      ICHECK=0
      M=N
      NN=N+1
      DO1I=1,M
      A(I,NN)=ATB(I)
      ID(I)=I
      DO1J=1,N
    1 A(I,J)=ATA(I,J)
      K=1
    2 NR=K
      NC=K
      B=DABS(A(K,K))
      DO4I=K,M
      DO4J=K,N
      IF(DABS(A(I,J)))4,4,3
    3 NR=I
      NC=J
      B=DABS(A(I,J))
    4 CONTINUE
      IF(NR-K)7,7,5
    5 DO6J=K,NN
      CC=A(NR,J)
      A(NR,J)=A(K,J)
    6 A(K,J)=CC
    7 CONTINUE
      IF(NC-K)10,10,8
    8 DO9I=1,M
    . CC=A(I,NC)
      A(I,NC)=A(I,K)
    9 A(I,K)=CC
      I=ID(NC)
      ID(NC)=ID(K)
      ID(K)=I
   10 CONTINUE
   11 IF(A(K,K))12,18,12
   12 KK=K+1
      DO14J=KK,NN
      A(K,J)=A(K,J)/A(K,K)
      DO 14I=1,M
      IF(K-I)13,14,13
   13 A(I,J)=A(I,J)-A(I,K)*A(K,J)
   14 CONTINUE
      K=KK
      IF(K-N)2,11,15
   15 CONTINUE
      DO 17I=1,N
      DO 17J=1,N
      IF(ID(J)-I)17,16,17
   16 QBW(I)=A(J,NN)
   17 CONTINUE
      GOTO20
   18 WRITE(6,19)
   19 FORMAT(10X,'NO UNIQUE SOLUTION')
      ICHECK=1
   20 RETURN
```

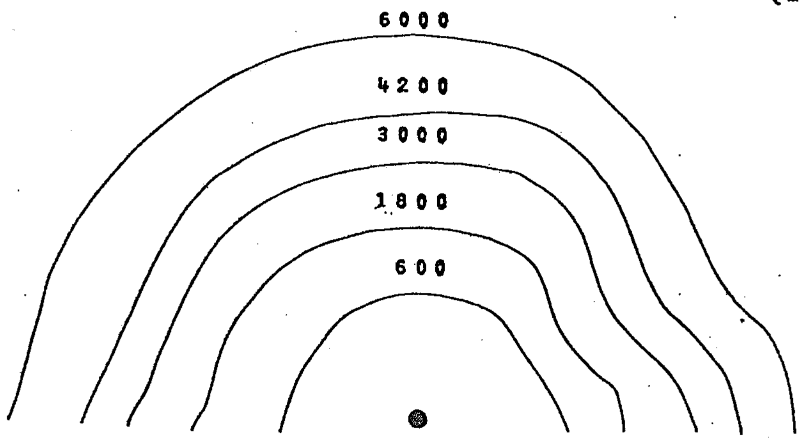
APPENDIX C

SELECTED EXPERIMENTAL RUNS

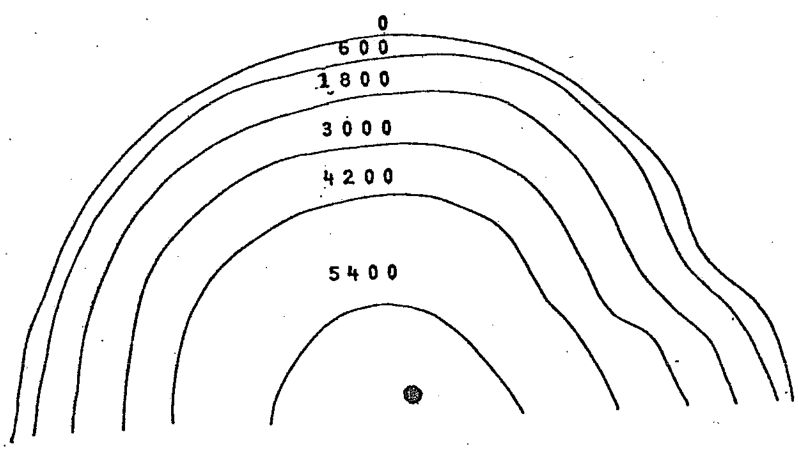
Run No.	Dip Angle Degrees	Density Difference gm/cc	Injection & Production Rates·cc/min	Gradient Ft/Mile	Store Time (sec.)	Bounded or Not Bounded
1	0	0	6	0	0	Not Bounded
2	0	0	6	2000	0	Not Bounded
3	0	0	6	2000	0	Bounded
4	15	.4	8	0	0	Not Bounded
5	15	.4	8	0	0	Bounded
6	30	.4	8	0	0	Not Bounded
7	30	.4	8	0	0	Bounded
8	30	.55	8	0	0	Not Bounded*
9	30	.55	8	0	0	Bounded*

Run 1

ϕ = 0°
Δp = 0
q = .1002 cc/sec
No Flux
(Time in Sec.)



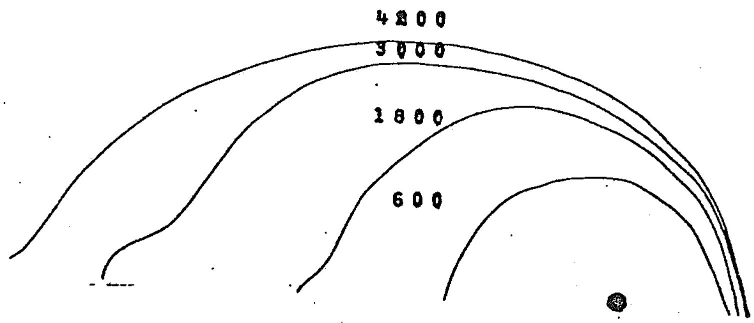
Injection



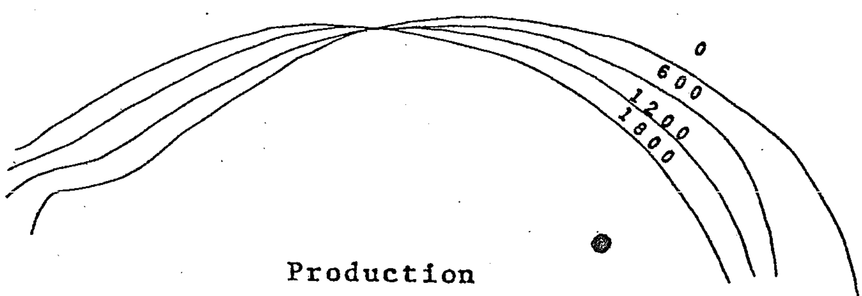
Production

Run 2

$\angle = 0°$
$\Delta\rho = 0$
$q = .1002$ cc/sec
Gradient = 2000 ft/mile
(Time in Sec.)



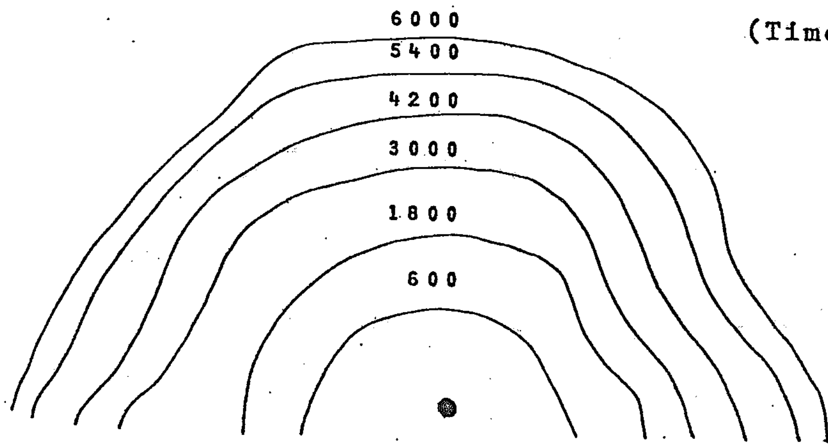
Injection



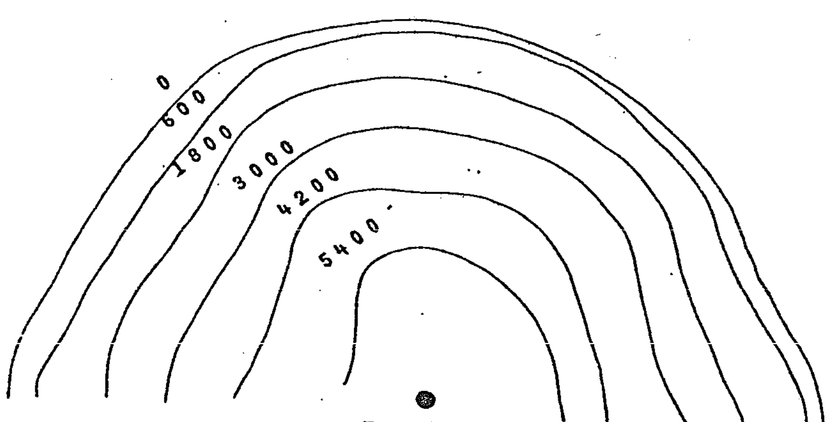
Production

Run 3

$\angle$ = 0°
$\Delta\rho$ = 0
q = .1002 cc/sec
Gradient = 2000 ft/mile
(Bounded)
(Time in Sec.)



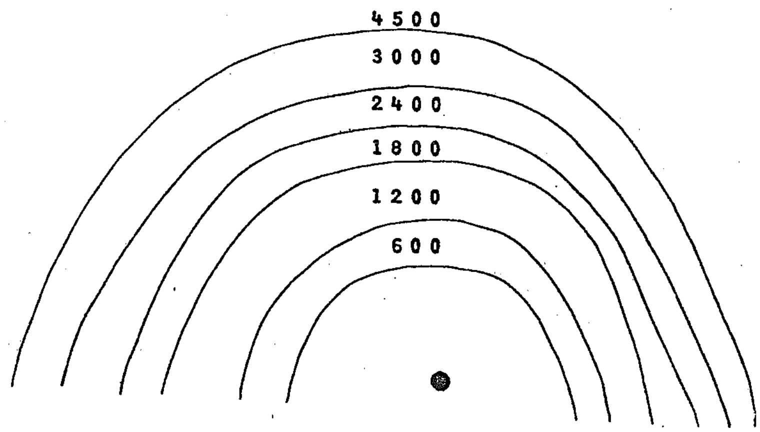
Injection



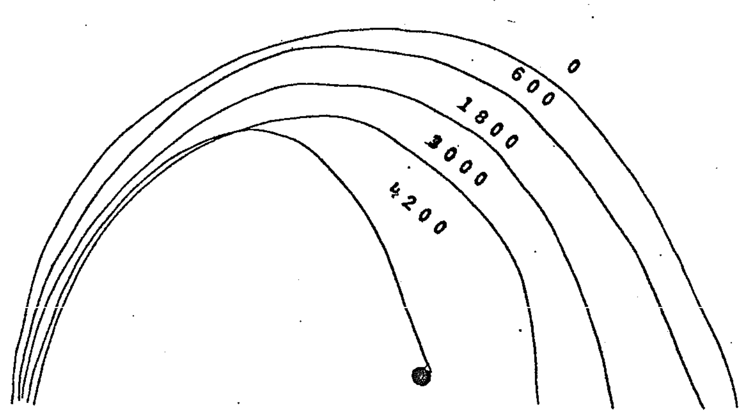
Production

Run 4

L = 15°
$\Delta\rho$ = .4 gm/cc
q = .1333 cc/sec
(Bounded)
(Time in Sec.)



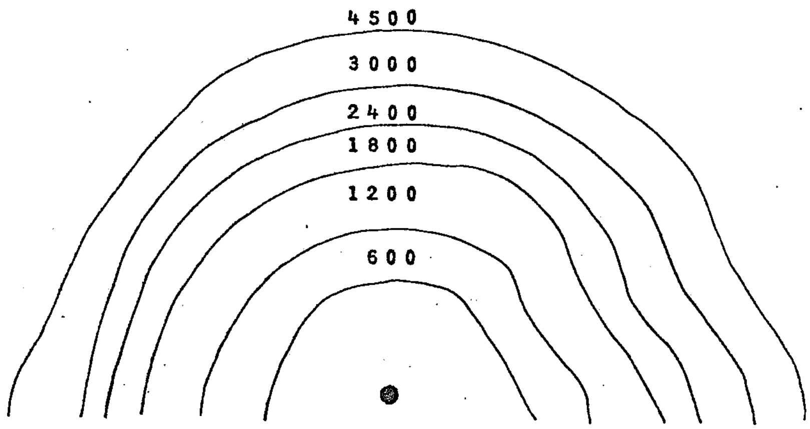
Injection



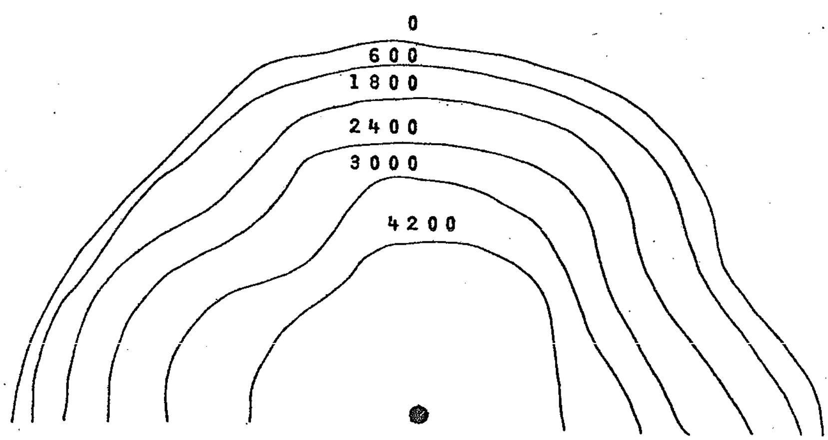
Production

Run 5

$\angle = 15°$
$\Delta\rho = .4$ gm/cc
$q = .1333$ cc/sec
(Bounded)
(Time in Sec.)



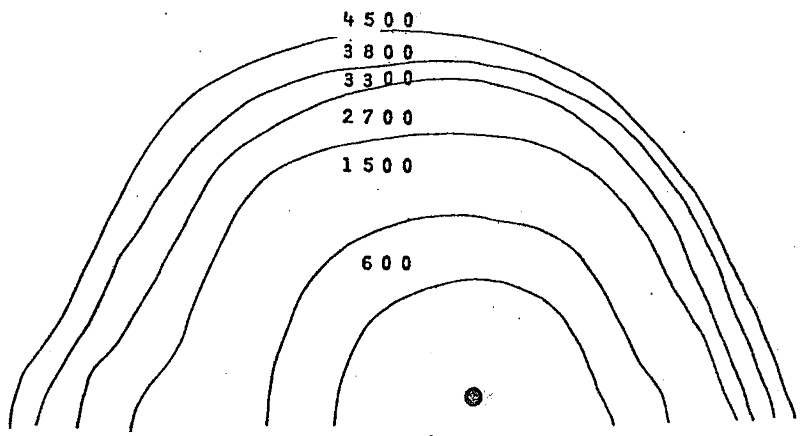
Injection



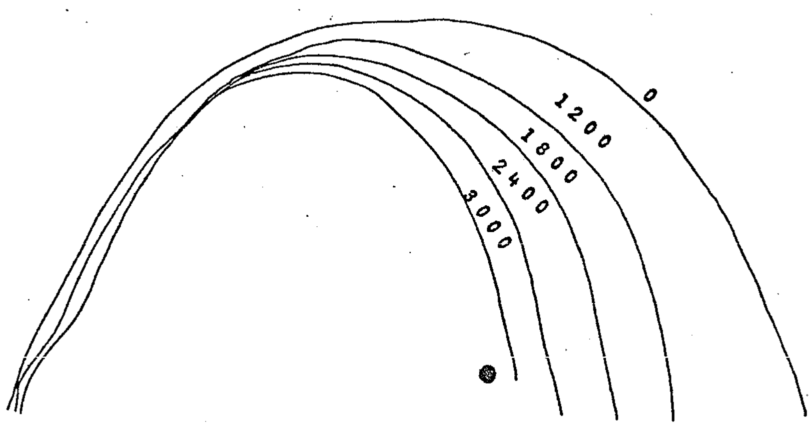
Production

Run 6

$\angle = 30°$
$\Delta\rho = .4$ gm/cc
$q = .1333$ cc/sec
(Time in Sec.)



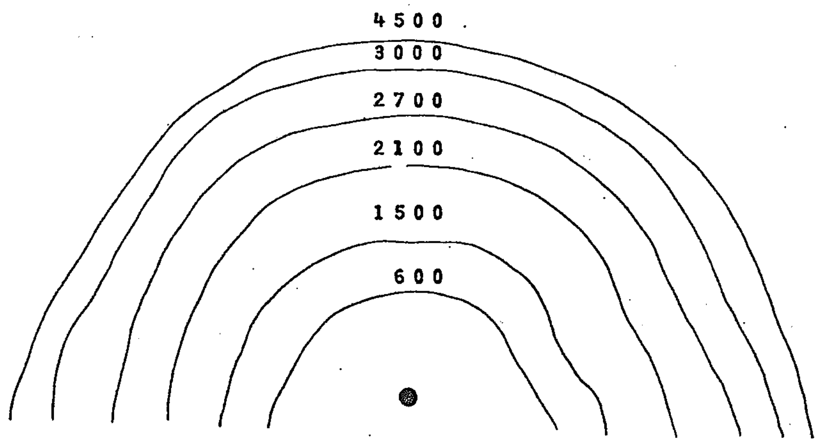
Injection



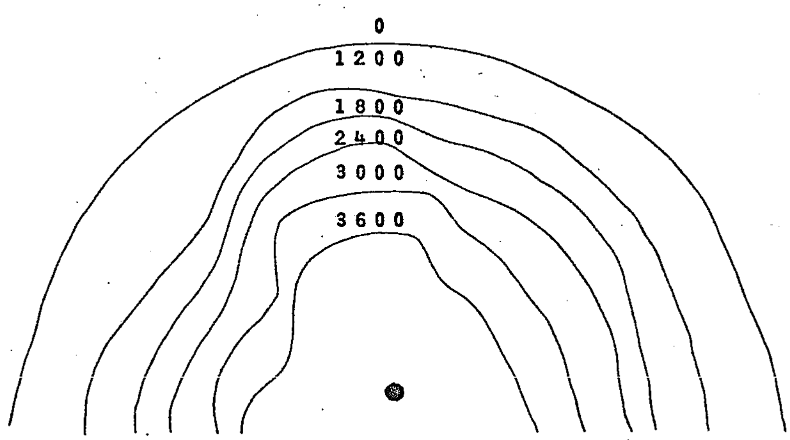
Production

Run 7

$\angle = 30°$
$\Delta\rho = .4$ gm/cc
$q = .1333$ cc/sec
(Bounded)
(Time in Sec.)



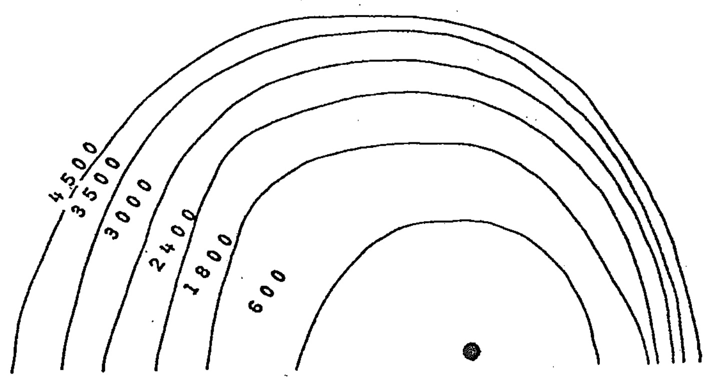
Injection



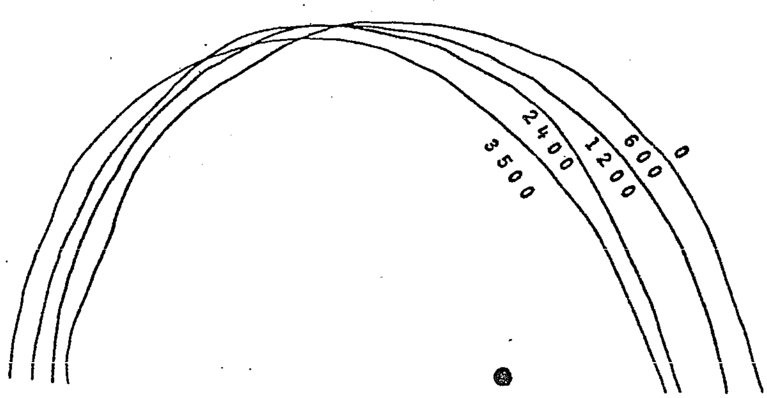
Production

Run 8

$\angle = 30°$
$\Delta\rho = .55$ gm/cc
$q = .1333$ cc/sec
(Time in Sec.)



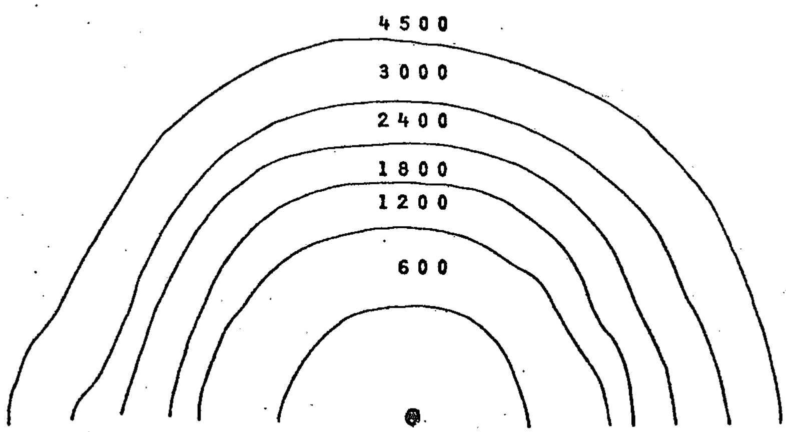
Injection



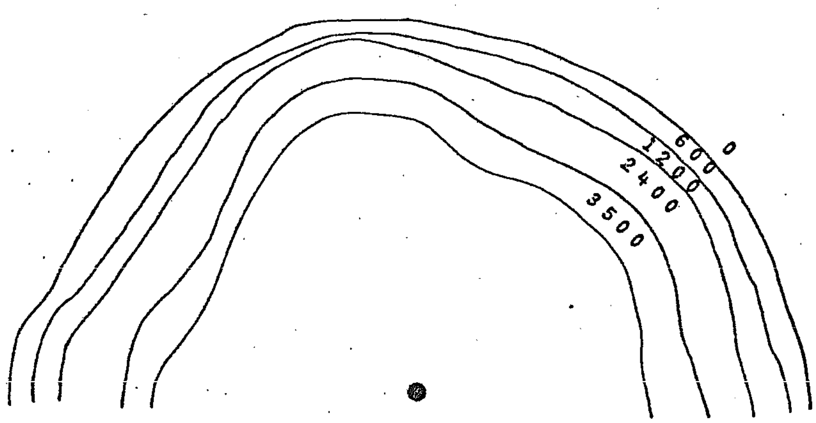
Production

Run 9

$\angle = 30°$
$\Delta\rho = .55$ gm/cc
$q = .1333$ cc/sec
(Bounded)
(Time in Sec.)



Injection



Production

VITA

Thomas Elfe Williams was born in Fort Belvoir, Virginia on July 6, 1954, the son of Col. and Mrs. G.W. Williams. After completing his work at Savanah Country Day School in Savanah, Georgia in May, 1972 he entered Clemson University at Clemson, South Carolina, from which he received a Bachelor of Science degree in Geology in May, 1976. In August, 1976, he enrolled in the Graduate School of Louisiana State University where he received a research assistantship to work toward a Master of Science degree in Petroleum Engineering.

He is married to the former Victoria Linne Gilbert of Piedmont, South Carolina, who is also a graduate of Clemson University.

EXAMINATION AND THESIS REPORT

Candidate: Thomas E. Williams

Major Field: Petroleum Engineering

Title of Thesis: Use of Bounding Wells to Counteract the Effects of Gravity in Dipping Aquifers.

Approved:

Walter R Whitehead

Major Professor and Chairman

James G. Traynham

Dean of the Graduate School

EXAMINING COMMITTEE:

Oscar K Kimbler

Raphael G Kazmann

Date of Examination:

April 7, 1978